Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION - 2017/2018

School of INSERT SCHOOL

BDIC2002J Discrete Mathematics

Time Allowed: 95 minutes

Instructions for Candidates

All questions carry equal marks. The distribution of marks in the right margin shown as a percentage gives an approximate indication of the relative importance of each part of the question.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

The Full Score of All Items of the Exam Paper

Item	1	2	3	4	5	6	7	Full
Full score	**14**	**16**	**14**	**14**	**14**	**14**	**14**	100

Obtained score

Question 1: **Suppose that R is a reflexive relation on the set A. Suppose that for arbitrary $a,b,c \in A$, if $(a,b) \in R$, $(a,c) \in R$, then $(b,c) \in R$ holds.**

Prove that R is an equivalence relation on A.

Obtained score

Question 2: **A square number is defined to be the square of some integer (e.g. 1,4,9,16 are square numbers). Let $A = \{1,2,3,\cdots,24,25\}$ be the set of all positive integers no greater than 25. Let R be a relation on A, defined by**

$(x, y) \in R$ iff xy is a square number.

(1) Prove that R is an equivalence relation on A.
(2) List all different equivalence classes (whose cardinality is greater than one) of elements of A.
(3) Give the cardinality of A/R (don't have to show details)

Obtained score

Question 3: **Compute the Principle Disjunctive Normal Form of**

$(P \wedge Q) \leftrightarrow R$

Obtained score

Question 4: **Compute the Prenex Normal Form of**

$\{\neg\exists x B(x) \vee \forall x\, [A(x) \to C(x, y)]\} \wedge \neg\forall y D(x, y)$

Obtained score

Question 5: **Let $G := \{(a,b)|a \in \mathbb{R} - \{0\}, b \in \mathbb{R}\}$. The operator $*$ is defined by $(a,b) * (c,d) = (ac, ad+b)$. Prove that $(G,*)$ is a group.**

Obtained score

Question 6: **Let $(G,*)$ be a group of order 50. Show that G has at least a subgroup of order 5.**

Obtained score

Question 7: **Prove that the complete bipartite graph $K_{3,3}$ is a non-planar graph.**