Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION - 2016/2017

BDIC2002J Discrete Mathematics

Time Allowed: 95 minutes

Instructions for Candidates

The distribution of marks in the right margin shown as a percentage gives an approximate indication of the relative importance of each part of the question.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

Answer ALL seven questions.

The Full Score of All Items of the Exam Paper

Item	1	2	3	4	5	6	7	Full
Full score	14	14	14	16	14	14	14	100

Obtained score

Question 1: **Let R be a binary relation on a set A. Suppose that R is reflexive and transitive. Prove that $R \circ R = R$**

Obtained score

Question 2: **Compute the Prenex Normal Form of**

$\forall x(A(x) \to B(x, y)) \leftrightarrow (\forall y C(y) \wedge \exists z D(y, z))$

Obtained score

Question 3: **Compute the Principle Conjunctive Normal Form of**

$(P \wedge Q) \vee \neg(Q \leftrightarrow R)$

Obtained score

Question 4: **A square number is defined to be the square of some integer (e.g. 1,4,9,16 are square numbers). Let $A = \{1, 2, 3, \cdots, 30, 31, 32\}$ be the set of all positive integers no greater than 32. Let R be a binary relation on A defined by**

$(x, y) \in R$ iff xy is a square number.

(1) Prove that R is an equivalence relation on A

(2) Give the cardinality of the quotient set A/R

(3) List all those elements of A/R whose cardinality is greater than two (write down them in the form of {a,b,c,…} instead of $[a]_R$)

Obtained score

Question 5: **Denote** $A = \mathbb{R} - \{2\}$ **to be the set of all real numbers except 2. Define the operator * by:** $a * b = ab - 2a - 2b + 6$**. Prove that (A,*) is a group.**

Obtained score

Question 6: **Let (G, *) be a non-commutative group. Suppose that G is a finite group. Prove that** $|G| \geq 6$

Obtained score

Question 7: **Let G be a tournament and let** $\{v_1, \ldots, v_n\}$ **be all the vertices of G, where** n **is the number of all vertices of G. Prove the following identity:**

$$\sum_{i=1}^{n}(\deg^+(v_i) - n)^2 - \sum_{i=1}^{n}(\deg^-(v_i))^2 = n^2$$