Semester One of Academic Year (2015---2016) of BJUT

《Discrete Mathematics》

Module Code: BDIC2002J

Exam Paper A

Exam Instructions: **Answer any SIX Items of all Questions**

Honesty Pledge:

I have read and clearly understand the Examination Rules of Beijing University of Technology and University College Dublin and am aware of the Punishment for Violating the Rules of Beijing University of Technology and University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I would accept the punishment thereof.

Pledger: ___________   **Class No:** ___________

BJUT Student ID: ___________   **UCD Student ID**___________

Notes:

The exam paper has 7 parts on 1 page, with a full score of 100 points. You are required to use the given Examination Book only.

Instructions for Candidates

Full marks will be awarded for complete answer to any **six** parts of all questions.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.

Total Score of the Exam Paper（For teachers' use only）

Item	1	2	3	4	5	6	7			Total Score
Full Score	16	17	16	17	17	17	17			
Obtained Score										

Obtained score

Part 1:

(1) Let R be a transitive relation on a set A. Prove that $R \circ R$ is also a transitive relation.

(2) Let R be a reflexive relation on a set A. Suppose that for any $(a,b) \in R, (b,c) \in R$, we always have $(c,a) \in R$. Prove that R is an equivalence relation on A.

Obtained score

Part 2: Let $A = \{1,2,3,4,5,6,7,8,9,10\}$ and let R be a relation on the set $A \times A$ defined by $(a,b)R(c,d)$ iff $a+d=b+c$

(1) Prove that R is an equivalence relation

(2) Compute the equivalence class $[(2,5)]_R$

Obtained score

Part 3:

(1) Compute the Principle Conjunctive Normal Form of

$$\neg\Big((P \to \neg Q) \to (Q \leftrightarrow \neg R)\Big).$$

(2) Compute the Prenex Normal Form of

$$\forall y\Big(\exists z A(x,y,z)) \vee \exists z B(x,z)\Big) \to \neg\exists x C(x,y)$$

Obtained score

Part 4: Let G be a finite group. Suppose that G is not a commutative group. Prove that the order of G is greater than five.

Obtained score

Part 5: Let $(G,*)$ be a group and let $(H,*)$, $(K,*)$ be two subgroup of $(G,*)$. We denote HK, KH to be the following two sets:

$$HK = \{h*k | h \in H, k \in K\}, \qquad KH = \{k*h | k \in K, h \in H\}$$

Prove that $(HK,*)$ is a subgroup of $(G,*)$ if and only if $HK = KH$

Obtained score

Part 6: Let G be a simple bipartite graph. Denote e to be the number of all edges of G and denote v to be the number of all vertices of G . Prove that

$$e \le \frac{1}{4}v^2 \text{ if } v \text{ is even;} \qquad e \le \frac{1}{4}(v^2-1) \text{ if } v \text{ is odd}$$

Obtained score

Part 7: Let G be a simple connected planar graph. Denote e to be the number of all edges of G and denote v to be the number of all vertices of G . Suppose that each face of G has at least k edges. Prove that $e \le \frac{k(v-2)}{k-2}$