BDIC Final_Exam_Template_22-23离散数学A卷

2024年6月3日 星期一 10:03

BDIC_Final_Exam_Template_22-23

Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION - 2022/2023

BDIC2002J/2025J Discrete Mathematics

Exam Test A

Time Allowed: 95 minutes

Instructions for Candidates

All items within each question carry equal marks. Detailed scores are given in the table.

BJUT Student ID: ______________ **UCD Student ID:** ______________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

Obtained score

Question 1: Single choice question (choose the only one item, fill the answer in bracket)

1.1 What is the function type of $f:\mathbb{Z}\times(\mathbb{Z}-\{0\})\to\mathbb{Q}$, $f(x,y)=x/y$ [A]

A、not an injection but a surjection B、not a surjection but an injection

C、a bijection D、neither an injection nor a surjection

1.2 What is the function type of $f:\mathbb{Z}\times\mathbb{Z}\to\mathbb{Z}$, $f(x,y)=x$ [A]

(1,1) → 1 , (1,2) → 1 => 不是 injection . 但满射

A、not an injection but a surjection B、not a surjection but an injection

C、a bijection D、neither an injection nor a surjection

1.3 What is the function type of $f:\mathbb{Z}\to\mathbb{Z}\times\mathbb{Z}$, $f(x)=(x,x)$ (1,3)无此值 [B]

A、not an injection but a surjection B、not a surjection but an injection

C、a bijection D、neither an injection nor a surjection

1.4 Which identity is wrong [ ]

$A\oplus B=(A\cap\overline{B})\cup(\overline{A}\cap B)$

A、$\overline{A\oplus B}=(A\cap\overline{B})\cup(\overline{A}\cap B)$ B、$A\oplus B\neq(A-B)\cup(\overline{A}\cap B)$

C、$\overline{A}\oplus B=A\oplus\overline{B}$ D、$A\oplus B=\overline{A}\oplus\overline{B}$

$\overline{A}\oplus B=(\overline{A}\cap\overline{B})\cup(A\cap B)$ $A\oplus\overline{B}=(\overline{A}\cap\overline{B})\cup(A\cap B)$

1.5 Let R, S be two equivalence relation on a finite set A. Which is an equivalence relation on A? [C]

A、$R-S$ B、$R\oplus S$ C、$R\cap S$ D、$R\cup S$

1.6 Which tautological implication is wrong? [ ]

A、$(P\to Q)\wedge\neg Q\Rightarrow\neg P$ B、$(P\to Q)\wedge(Q\leftrightarrow R)\Rightarrow R\to P$

C、$(P\leftrightarrow Q)\wedge(Q\to R)\Rightarrow\neg R\to\neg P$ D、$(P\leftrightarrow Q)\wedge(Q\to R)\Rightarrow Q\to P\wedge R$

1.7 Which tautological implication is wrong? [ ]

A、$\forall x\forall yP(x,y)\Rightarrow\exists y\forall xP(x,y)$ B、$\neg\forall y\exists xP(x,y)\Rightarrow\neg\exists x\forall yP(x,y)$

C、$\neg\exists x\forall yP(x,y)\Rightarrow\neg\forall y\exists xP(x,y)$ D、$\forall x\exists yP(x,y)\Rightarrow\exists y\exists xP(x,y)$

1.8 Which tautological implication is wrong? [ ]

A、$\forall x(A(x)\to B(x))\Rightarrow\forall xA(x)\to\exists xB(x)$ B、$\exists xA(x)\to\forall xB(x)\Rightarrow\forall x(A(x)\to B(x))$

C、$\forall x(A(x)\leftrightarrow B(x))\Rightarrow\exists xA(x)\leftrightarrow\forall xB(x)$ D、$\forall x(A(x)\leftrightarrow\neg B(x))\Rightarrow\forall xA(x)\leftrightarrow\neg\exists xB(x)$

1.9 G is the three-order directed complete graph. G' is a subgraph of G obtained by deleting an edge from G. Choose the graph type of G'. [ ]

A、strongly connected graph B、not strongly connected graph but unilaterally connected graph

C、not unilaterally connected graph but weakly connected graph　D、none of the above

1.10 Choose the graph type of K_6 [ ]

A、Eulerian graph　B、semi-Eulerian graph　C、Hamiltonian graph
D、bipartite graph　E、planar graph

Obtained score

Question 2: Multiple choice question (choose at least two items, fill the answer in bracket)

2.1 The principle disjunctive normal form of $(P \to Q) \land (P \to \neg R)$ consists of [BDFGH]

A、$P \land Q \land R$　B、$P \land Q \land \neg R$　C、$P \land \neg Q \land R$　D、$\neg P \land Q \land R$
E、$P \land \neg Q \land \neg R$　F、$\neg P \land Q \land \neg R$　G、$\neg P \land \neg Q \land R$　H、$\neg P \land \neg Q \land \neg R$

2.2 The principle disjunctive normal form of $(P \to \neg Q) \leftrightarrow (P \to R)$ consists of [BCDFGH]

A、$P \land Q \land R$　B、$P \land Q \land \neg R$　C、$P \land \neg Q \land R$　D、$\neg P \land Q \land R$
E、$P \land \neg Q \land \neg R$　F、$\neg P \land Q \land \neg R$　G、$\neg P \land \neg Q \land R$　H、$\neg P \land \neg Q \land \neg R$

2.3 The principle disjunctive normal form of $(P \to \neg Q) \land (P \leftrightarrow R)$ consists of C [F H]

A、$P \land Q \land R$　B、$P \land Q \land \neg R$　C、$P \land \neg Q \land R$　D、$\neg P \land Q \land R$
E、$P \land \neg Q \land \neg R$　F、$\neg P \land Q \land \neg R$　G、$\neg P \land \neg Q \land R$　H、$\neg P \land \neg Q \land \neg R$

2.4 The principle conjunctive normal form of $(P \to Q) \land (P \leftrightarrow R)$ consists of B D E F G [ ]

A、$P \lor Q \lor R$　B、$P \lor Q \lor \neg R$　C、$P \lor \neg Q \lor R$　D、$\neg P \lor Q \lor R$
E、$P \lor \neg Q \lor \neg R$　F、$\neg P \lor Q \lor \neg R$　G、$\neg P \lor \neg Q \lor R$　H、$\neg P \lor \neg Q \lor \neg R$

2.5 The principle conjunctive normal form of $Q \land \neg(P \leftrightarrow \neg R)$ consists of A B D E F [G]

A、$P \lor Q \lor R$　B、$P \lor Q \lor \neg R$　C、$P \lor \neg Q \lor R$　D、$\neg P \lor Q \lor R$
E、$P \lor \neg Q \lor \neg R$　F、$\neg P \lor Q \lor \neg R$　G、$\neg P \lor \neg Q \lor R$　H、$\neg P \lor \neg Q \lor \neg R$

2.6 The principle conjunctive normal form of $(P \land Q) \lor (\neg P \leftrightarrow R)$ consists of A C [F]

A、$P \lor Q \lor R$　B、$P \lor Q \lor \neg R$　C、$P \lor \neg Q \lor R$　D、$\neg P \lor Q \lor R$
E、$P \lor \neg Q \lor \neg R$　F、$\neg P \lor Q \lor \neg R$　G、$\neg P \lor \neg Q \lor R$　H、$\neg P \lor \neg Q \lor \neg R$

2.7 Let R be a binary relation on $\mathbb{Z}^+$ defined by xRy iff $x \times (y-1) = 8$. Choose the relation types of R.
[(1,9), (2,5), (4,3), (8,2)]

A、reflexive　B、antireflexive　C、symmetric　D、antisymmetric　E、transitive

B, D

2.8 Let R be a binary relation on $\mathbb{Z}^+$ defined by xRy iff $3|(x+y)$ or $3|(x-y)$. Choose the relation types of R. 加法对称, [AC]

(3,3)　(3,6), (6,3)

A、reflexive　B、antireflexive　C、symmetric　D、antisymmetric　E、transitive

2.9 Let R,S be two binary relations on $\mathbb{Z}^+$ defined by xRy iff $x < y$; xSy iff $x|y$. Choose the relation types of $R - S$. [ ]

A、reflexive　B、antireflexive　C、symmetric　D、antisymmetric　E、transitive

2.10 Let R be a binary relation on $\mathbb{Z}^+$ defined by xRy iff $x \times y = x + y$. Choose the relation types of R.
$R = \{(2,2)\}$ [CDE]

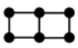
A、reflexive　B、antireflexive　C、symmetric　D、antisymmetric　E、transitive

2.11 Let G be an undirected graph, with vertex number v and edge number e. Which are sufficient and necessary conditions such that G is a tree? [ ]

A、G is connected and has no loop　B、G is connected and $e = v - 1$
C、G has no circle and $e = v - 1$　D、G is connected and has no bridge

2.12 G is the three-order directed complete graph. G' is a subgraph of G obtained by deleting three edges from G. Choose the graph type of G'. [ ]

A、strongly connected graph　B、not strongly connected graph but unilaterally connected graph
C、not unilaterally connected graph but weakly connected graph　D、not weakly connected graph

2.13 Choose the graph type of  [ ]

A、Eulerian graph　B、semi-Eulerian graph　C、Hamiltonian graph
D、bipartite graph　E、planar graph　F、non-planar graph

2.14 Choose the graph type of  [ ]

A、Eulerian graph　B、semi-Eulerian graph　C、Hamiltonian graph
D、bipartite graph　E、planar graph　F、non-planar graph

2.15 Choose the graph type of $K_{3,4}$ [ ]

A、Eulerian graph　B、semi-Eulerian graph　C、Hamiltonian graph
D、semi-Hamiltonian graph　E、bipartite graph　F、non-planar graph

2.9 R,S 定义在 Z^+, xRy 若 $x<y$

xSy 若 $x|y$

$\therefore R = \{(1,2)(1,3),(1,4),(2,3),(2,4),(3,4)$

$S = \{(1,1),(2,2),(3,3),(1,2),(1,3),(2,4)\}$

判断:

R−S (小于又不能整除)

反自反, 反对称,

不传递　$(2,3),(3,8) \in R-S$

$(2,8) \notin R-S$

2.10

Obtained score

Question 3: Judgement question (fill **T** (true) or **F** (false) in bracket)

3.1 For three sets A, B, C, if $A \in B, B \subseteq C$, then $A \subseteq C$ [F]

3.2 For three sets A, B, C, $(A \cap B) \oplus (A \cap C) = (A - B) \oplus (A - C)$ [T]

3.3 For any binary relation R on a finite set A, $R \circ R^{-1}$ is always a symmetric relation on A, where $\circ$ denotes the composition of two relations. [T]

3.4 For proposition formulae A, B, $(A \leftrightarrow B) \to (A \lor B) \Leftrightarrow A \land B$ [F]

3.5 For proposition formulae P, Q, R, S, $(P \lor Q) \land (P \to R) \land (Q \to S) \Rightarrow \neg R \to S$ [T]

3.6 For proposition functions $A(x), B(x)$, $\exists x(A(x) \to B(x)) \Leftrightarrow \exists x A(x) \to \forall x B(x)$ [F]

3.7 For proposition functions $P(x), Q(x)$, $\forall x P(x) \lor \exists x Q(x) \Rightarrow \exists x(P(x) \lor Q(x))$ [T]

3.8 Let G be a simple connected graph. Then its complement $\bar{G}$ is always an unconnected graph. [ ]

3.9 Let G be an n-order undirected simple graph. Suppose $\deg(x) + \deg(y) \geq n$ holds for any two vertices $x \neq y$ of G. Then G is a Hamiltonian graph. [ ]

3.10 Let G be a simple connected planar graph with at least a finite face. Then $3v - e \geq 6$, where v is the vertex number and e is the edge number. [ ]

Obtained score

Question 4: fill-in-the-blank question

4.1 The prenex normal form of $\neg\forall x A(x) \land (\forall x B(x) \to \exists y C(x,y))$ is ______________.

4.2 The prenex normal form of $\forall x A(x) \leftrightarrow \forall x B(x)$ is ______________.

4.3 The prenex normal form of $\neg\exists x \forall y(A(x,y) \to B(y))$ is ______________.

$(A \cap \bar{B}) - (A \cap \bar{C})$

3.2 left =

$[(A \cap B) - (A \cap C)] \cup [(A \cap C) - (A \cap B)]$

$= (A \cap B \cap \bar{C}) \cup (A \cap C \cap \bar{B})$

right $= [(A-B)-(A-C)] \cup [(A-C)-(A-B)]$

$= (A \cap \bar{B} \cap C) \cup (A \cap \bar{C} \cap B)$

Question 4: fill-in-the-blank question

4.1 The prenex normal form of $\neg\forall x A(x) \wedge (\forall x B(x) \rightarrow \exists y C(x,y))$ is ______.

4.2 The prenex normal form of $\forall x A(x) \leftrightarrow \forall x B(x)$ is ______.

4.3 The prenex normal form of $\neg\exists x\forall y(A(x,y) \rightarrow B(y))$ is ______. 幂集?

4.4 For arbitrary two sets A, B, let P denote the power set. Then $P(A-B)$ $\subseteq$ $P(A)-P(B)$ (fill one symbol of "$=, \neq, \subseteq$ or $\supseteq$")

$A:\{1,2,5\}$ $B:\{1,5\}$ $P(A-B)=\{\emptyset,\{2\}\}$

4.5 Let R be the divide relation on $A=\{2,3,4,5,6,8,10\}$, i.e., xRy iff $y=mx$ for some integer m. Then the greatest element of the poset (A,R) is null, the least element of the poset (A,R) is null. The maximal elements of the poset (A,R) are 6, 8, 10, the minimal elements of the poset (A,R) are 2, 3, 5.

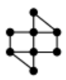
4.6 Let R be a binary relation on $A=\{1,2,3,\ldots,15,16\}$ (i.e., A is the set of all positive integers ≤ 16) defined by xRy iff $x\times y$ is a *square number* (i.e., 1,4,9,16,25,36,49,…). Then R is an equivalence relation on A. The cardinality of the quotient set A/R is 11 (势). For any $x \in A$, $|[x]_R| \leq$ 4.

4.7 Let A be a set with $|A|=3$. Let R be the set of all antireflexive relations on A, and let S be the set of all antisymmetric relations on A. Then $|R \cap S| =$ 27.

4.8 Let A be a set with $|A|=3$. Let R be the set of all reflexive relations on A, and let S be the set of all symmetric relations on A. Then $|R \cup S| =$ 253.

4.9 Let $A=\{1,2\}$ be a set and let P denote the power set. There are 24 different bijections from $P(A)$ to $P(A)$. There are ____ different surjections from $P(A)$ to A.

4.10 Let $A=\{1,2,3,4,5,6,7\}$ be a set, and let R be a binary relation on $A\times A$ defined by $(a,b)R(c,d)$ iff $a+d=b+c, \forall(a,b),(c,d) \in A\times A$. Then R is an equivalence relation, and $[(1,6)]_R = \{(2,7),(1,6)\}$

4.11 Let $G =$ . Then the length of G' longest circle is ____. The length of G's longest trail is ____.

4.12 K_4 has _____ faces with odd degree. $K_{2,4}$ has _____ faces with even degree.

4.13 Let G be an undirected graph with twelve edges. If G has three 2-degree vertices, two 4-degree vertices, and other vertices have odd degrees. Then ____ $\leq |G| \leq$ ____, where $|G|$ denotes G's order.

4.14 After we delete at least ____ vertices from $K_{5,6}$, it becomes a graph which is both a Eulerian graph and a Hamiltonian graph.

4.15 Let G be a tournament whose vertex-set is $\{v_1, v_2, \ldots, v_n\}$. Then

$\sum_{i=1}^{n}(\deg^+(v_i)+n)^2 - \sum_{i=1}^{n}(\deg^-(v_i))^2 =$ ______.

幂集

4.6

$[1]_R=\{1,4,9,16\}$
$[2]_R=\{2,8\}$
$[3]_R=\{3,12\}$
$[5]_R=\{5\}$
$[6]_R=\{6\}$
$[7]_R=\{7\}$
$[10]_R=\{10\}$
$[11]_R=\{11\}$
$[13]_R=\{13\}$
$[14]_R=\{14\}$
$[15]_R=\{15\}$

4.7 $|A|=3$

$R=$ A上所有反自反关系，

$S=$ A上所有反对称

$R\cap S=$ A上即反自反又反对称

$3\times3\times3$

$(x_{12},x_{21}) \in (1,0),(0,1),(0,0)$,

$(0,1),(1,0)$

$2\times2\times2\times3\times3\times3$

8×27

$27\times8=216$

0 1 2
0
1
2

4.8 $|R\cup S| = |R| + |S| - |R\cap S|$

2^6 64

216

27

$280-27=253$

4.9 $A=\{1,2\}$, $P(A)$ 到 $P(A)$ ___ 双射

$P(A)=4$ $P(A)$ 到 A ___ 满射