# Beijing-Dublin International College

**SEMESTER I FINAL EXAMINATION - 2016/2017**

**School of INSERT SCHOOL**：Beijing University of Technology

**MODULE CODE and MODULE TITLE**

BDIC2005J，Probability and Statistics

**HEAD OF SCHOOL NAME**：BDIC

**MODULE COORDINATOR NAME**：Xie Tianfa，Zhao Xu

**Time Allowed: 90 minutes**

**Instructions for Candidates**

All questions carry equal marks. The distribution of marks in the right margin shown as a percentage gives an approximate indication of the relative importance of each part of the question.

**BJUT Student ID: ________________** **UCD Student ID: ________________**

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

**Honesty Pledge**：**____________________________________________ (Signature)**

**Instructions for Invigilators**

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

| Obtained score |
|---|
| |

**Question 1:**
**Vacancy (Each blank 3 marks)**

(1) There are two events $A, B$. $P(A)=0.5$, $P(B)=0.5$, $P(A\cup B)=0.7$, then $P(A|B)=$______ .

(2) Let $X$ be random variable. $E(X)=7$, $D(X)=5$. Use Chebyshev inequality to find $P\{3<X<11\}\geq$______.

(3) Suppose that the two-dimensional random variable ($X$, $Y$) follows a uniform distribution on R={($x$, $y$) | $0<x<y<1$}, then the correlation coefficient of $X$ and $Y$ is $\rho$=_____.

(4) Let $X_1,\cdots,X_n$ be a sample from $N(\mu,\sigma^2)$.

$$\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i,\quad S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2.$$

$$\bar{X}\sim\_\_\_\_\_\_,\ \sqrt{n}(\bar{X}-\mu)/\sqrt{S^2}\sim\_\_\_\_\_\_,\ E(S^2)=\_\_\_\_\_\_.$$

(5) Consider the data on the percentage of sugar content of a standard variety of sugar bee. Now taken 10 samples from the sugar bee, the sample mean is 18.13, the sample standard deviation is 0.46. Under the assumptions of the normal one-sample model, determine the 95% confidence interval for the mean percentage of sugar content of the standard variety [________ , _________] and the 95% confidence interval for the variance percentage of sugar content of the standard variety [________ , _________].

| Obtained score |
|---|
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**Question 2: (10 marks)**
A radio station sends signals "-" and "." in probabilities 0.6 and 0.4, respectively, for some interruptions when the radio station sends ".", the accepting station unnecessarily receive the signal ".", but it receives signals "." and "-" in probabilities 0.8 and 0.2, respectively. Meanwhile, when the radio station sends "-", the accepting station may receive signals "-" and "." in probabilities 0.9 and 0.1, respectively.

(a) What is the probability that the accepting station receives the signal "."?

(b) If the accepting station receives ".", what is the probability that the radio station sends signal "."?

| Obtained score |
|---|
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**Question 3: (15 marks)**
Suppose that the probability density function of the continuous random variable $X$ is

$$f(x)=\begin{cases}\dfrac{2x}{\pi^2}, & 0<x<\pi\\ 0, & \text{elsewhere}\end{cases}$$

Let $Y=\sin X$. Find:

(a) $f_Y(y)$, the probability density function of $Y$;

(b) $E(X)$;

(c) $D(X)$.

| Obtained score |
|---|
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**Question 4: (15 marks)**

Suppose that the joint probability density of two random variables *X* and *Y* are given by

$$f(x,y)=\begin{cases}1, & |y|<x, 0<x<1,\\ 0, & \text{elsewhere.}\end{cases}$$

Find:

(a) the marginal densities $f_X(x)$, $f_Y(y)$;

(b) whether the two random variables *X* and *Y* are independent or not (Please give your reason);

(c) $E(X)$, $E(Y)$.

| Obtained score |
|---|
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**Question 5: (15 marks)**

Suppose $X_1, X_2 \cdots X_n$ is a sample from *X*, and the probability density function of *X* is

$$f(x;\mu,\sigma^2)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}},\ -\infty<x<\infty,$$

where $\sigma>0$. Find:

(a) the moment estimators $\hat{\mu},\hat{\sigma}^2$ of $\mu,\sigma^2$ ;

(b) the maximum likelihood estimators $\mu^*,\sigma^{2*}$ of $\mu,\sigma^2$;

(c) whether the moment estimators $\hat{\mu},\hat{\sigma}^2$ are unbiased or not (Please give your reason).

| Obtained score |
|---|
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**Question 6: (15 marks)**

A kind of product, the standard length is 31.5 mm, and the length of this kind of product *X* is assumed to follow normal distribution $N(\mu,\sigma^2)$. Now taken 10 from the products, the sample mean is 30, the sample standard deviation is 2.

Question: at level of significance 0.05,

(a) can we accept $\mu$= 31.5？

(b) can we accept $\sigma$=1.5？

The *t* distribution table and the $\chi^2$ distribution table

| | | | |
|---|---|---|---|
| $t_9(0.025)=2.2622$ | $t_9(0.05)=1.8331$ | $t_{10}(0.025)=2.2281$ | $t_{10}(0.05)=1.8125$ |
| $\chi_9^2(0.025)=19.023$ | $\chi_9^2(0.05)=16.919$ | $\chi_9^2(0.975)=2.700$ | $\chi_9^2(0.95)=3.325$ |
| $\chi_{10}^2(0.025)=20.483$ | $\chi_{10}^2(0.05)=18.307$ | $\chi_{10}^2(0.975)=3.247$ | $\chi_{10}^2(0.95)=3.940$ |

## Appendix :