Semester One of Academic Year (2015---2016) of BJUT

《 Probability and Statistics》

Module Code: ____BDIC2005J____

Exam Paper A

Exam Instructions: __Answer ALL Questions__________________________________

Honesty Pledge:

I have read and clearly understand the Examination Rules of Beijing University of Technology and University College Dublin and am aware of the Punishment for Violating the Rules of Beijing University of Technology and University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I would accept the punishment thereof.

Pledger: ___________   **Class No:** ___________

BJUT Student ID: ___________   **UCD Student ID**___________

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Notes：

The exam paper has 2 parts on 4 pages, with a full score of 100 points. You are required to use the given Examination Book only.

Instructions for Candidates

Full marks will be awarded for complete answer to **All** questions.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.

Obtained score

Part 1: Vacancy (Each blank 3 marks)

(1) There are two events A, B. $P(A) = 0.5$, $P(B) = 0.6$, $P(A \cup B) = 0.7$,

$P(A|B) =$ ___________.

(2) If X and Y are independent, $X \sim N(3,\ 3^2)$, $Y \sim N(1,\ 2^2)$, let $Z = X - 2Y$,

$Z \sim$ ___________, $P\{-4 < Z < 6\} =$ _____________. ($\Phi(1)=0.8413$ $\Phi(2)=0.9772$)

(3) Suppose X follows a poisson distribution with parameters λ, denote $X \sim P(\lambda)$.

If $P\{X \geq 1\} = 1 - e^{-2}$, $\lambda =$ __________, $P\{X = 1\} = =$ ____________.

(4) Let $X_1, X_2, \cdots, X_n (n > 2)$ be a sample from $N(\mu, \sigma^2)$.

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 \cdot$$

$\sqrt{n}(\overline{X} - \mu)/\sigma \sim$ __________, $\sqrt{n}(\overline{X} - \mu)/\sqrt{S^2} \sim$ __________, $(n-1)S^2/\sigma^2 \sim$ __________.

(5) A random sample of 10 CitiBank VISA cardholder accounts indicated a sample mean debt of $1,220 with a sample standard deviation of $840. Construct a 95 percent confidence interval estimate of the average debt of all cardholders (assuming normal population)

[______________, ______________].

The t distribution table and the χ^2 distribution table

$t_9(0.025) = 2.2622$	$t_9(0.05) = 1.8331$	$t_{10}(0.025) = 2.2281$	$t_{10}(0.05) = 1.8125$
$\chi_9^2(0.025) = 19.023$	$\chi_9^2(0.05) = 16.919$	$\chi_9^2(0.975) = 2.700$	$\chi_9^2(0.95) = 3.325$
$\chi_{10}^2(0.025) = 20.483$	$\chi_{10}^2(0.05) = 18.307$	$\chi_{10}^2(0.975) = 3.247$	$\chi_{10}^2(0.95) = 3.940$

Obtained score

Part 2: Calculation (Show all your answer in detail)

(1) Suppose that an insurance company classifies people into one of three classes — good risks, average risks, and bad risks. Their records indicate that the probabilities that good, average, and bad risk persons will be involved in an accident over a 1-year span are, respectively, 0.05, 0.15, and 0.30. If 20 percent of the population are "good risks," 50 percent are "average risks," and 30 percent are "bad risks."

(a) What proportion of people have accidents in a fixed year?

(b) If policy holder A had no accidents in a fixed year, what is the probability that he or she is a good risk?

(2) Suppose that the probability density function of the continuous random variable X is

$$f(x)=\begin{cases}1-|x|, & x\in(-1,\ 1)\\ 0, & elsewhere.\end{cases}$$

Let $Y=X^2$.

Find: (a) $f_Y(y)$, the probability density function of Y; (b) $P\{0.5<Y<2\}$; (c) E(Y) and D(Y).

(3) Suppose that the joint probability density of two random variables *X* and *Y* are given by

$$f(x,y)=\begin{cases} c\cdot e^{-x}, & 0\le y\le x<\infty \\ 0, & elsewhere. \end{cases}$$

Find: (a) the value of constant *c*; (b) the marginal densities $f_X(x)$, $f_Y(y)$;

(c) whether the two random variables *X* and *Y* are independent or not.(Please give your reason).

(4) Suppose $X_1, X_2 \cdots X_n$ is a sample from *X*, and the probability density function of *X* is

$$f(x;\theta) = \begin{cases} \sqrt{\theta}\ e^{-\sqrt{\theta}x}, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

where $\theta > 0$. Find:

(a) the moment estimator of θ ;

(b) the maximum likelihood estimator of θ.

(5) The capacities (in ampere-hours) of 10 batteries were recorded as follows:

140,136,150,144,148,152,138,141,143,151

Under the assumption that the capacity is normal distributed as $N(\mu, \sigma^2)$.

Question: at level of significance 0.05,

(a) test $H_0 : \mu = 143 \leftrightarrow H_1 : \mu \neq 143$, (b) test $H_0 : \sigma = 5 \leftrightarrow H_1 : \sigma \neq 5$.