Semester One of Academic Year (2014---2015) of BJUT

《 Probability and Statistics 》 Exam Paper A

Exam Instructions: __Answer ALL Questions__

Honesty Pledge:

I have read and clearly understand the Examination Rules of Beijing University of Technology and University College Dublin and am aware of the Punishment for Violating the Rules of Beijing University of Technology and University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I would accept the punishment thereof.

Pledger: ___________ **Class No:** ___________

BJUT Student ID: ___________ **UCD Student ID**___________

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Notes：

The exam paper has__2__ parts on __8__ pages, with a full score of 100 points. You are required to use the given Examination Book only.

Total Score of the Exam Paper（For teachers' use only）

Item	Part 1	Part 2 (1)	Part 2 (2)	Part 2 (3)	Part 2 (4)	Part 2 (5)	Total Score
Full Score	30	14	14	14	14	14	
Obtained Score							

Obtained score

Part 1: Vacancy (Each blank 3 marks)

(1) There are two events A, B. $P(A)=0.5$, $P(B)=0.6$, $P(A\cup B)=0.7$. $P(B|A)=$________.

(2) Let random variable X_1, X_2 be mutually independent, $X_1 \sim N(3,\ 3^2)$, $X_2 \sim N(1,\ 2^2)$,

$X = X_1 - 2X_2$, $X\sim$________, $P\{-4<X<6\}=$________.

Note: $\Phi(x)$ is distribution function of normal distribution $N(0,\ 1)$, $\Phi(1)=0.8413$,

$\Phi(2)=0.9772$.

(3) Let random variable X follow a uniform distribution U(a, b), $E(X)=$_____, $D(X)=$_____.

(4) Let $X_1, X_2, \cdots, X_n (n>2)$ be a sample from $N(\mu,\sigma^2)$.

$$\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i,\quad S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\overline{X})^2 .$$

$\sqrt{n}(\overline{X}-\mu)/\sigma\sim$________, $\sqrt{n}(\overline{X}-\mu)/\sqrt{S^2}\sim$________, $(n-1)S^2/\sigma^2\sim$________.

(5) Let $X_1, \cdots, X_n$ be a sample from $N(\mu,\sigma^2)$. μ and σ^2 are unknown parameters.

The confidence interval of μ with confidence level 95% is [__________, __________].

The confidence interval of σ^2 with confidence level 95% is [__________, __________].

Obtained score

Part 2: Calculation (Show all your answer in detail)

(1) Suppose that in a class, 45%, 30%, 25% of the students majoring in mechanical engineering, electrical engineering and civil engineering, respectively. In the final exam, 20% of students major in mechanical engineering, 25% of students major in electrical engineering, 10% of students major in civil engineering got grade A.

(a) Select a student randomly from this class, what is the probability that he(she) got an A in the exam?

(b) Select a student who got an A at random, what is the probability that he(she) is major in civil engineering?

Obtained score

(2) Suppose that the probability density function of the continuous random variable X is

$$f_X(x)=\begin{cases} x/8, & 0<x<4, \\ 0, & \text{elsewhere} . \end{cases}$$

Let $Y = 2X+8$. Find:

(a) the probability density function of Y $f_Y(y)$; (b) $P\{9<Y<10\}$; (c)E(Y) and D(Y).

Obtained score

(3) Suppose that the joint probability density of two random variables are given by

$$f(x,y)=\begin{cases} cy\ (2-x), & 0\le x\le 1,\ \ 0\le y\le x, \\ 0, & \text{elsewhere.} \end{cases}$$

Find: (a) constant *c*; (b) the marginal densities $f_X(x),\ \ f_Y(y)$;

(c) whether the two random variables *X* and *Y* are independent or not.(Please give your reason)

Obtained score

(4) Suppose $X_1, X_2 \cdots X_n$ is a sample from *X*, the probability density function is

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x \le 0. \end{cases}$$

where $\lambda > 0$. Find:

(a) the moment estimator of λ;

(b) the maximum likelihood estimator of λ.

Obtained score

(5) A company claims that the average life of a certain type of battery is 21.5 hours, and the life of this kind of battery X is assumed to follow normal distribution $N(\mu, \sigma^2)$. A laboratory tests batteries manufactured by this company. Now taken 25 from the batteries, the sample mean is 20.0, the sample variance is 10.0.

Question: at level of significance 0.05,

(a) test $H_0 : \mu = 21.5 \leftrightarrow H_1 : \mu \neq 21.5$, (b) test $H_0 : \sigma = 3.5 \leftrightarrow H_1 : \sigma \neq 3.5$.

The *t* distribution table and the χ^2 distribution table

$t_{24}(0.025) = 2.0639$	$t_{24}(0.05) = 1.7109$	$t_{25}(0.025) = 2.0595$	$t_{25}(0.05) = 1.7081$
$\chi^2_{24}(0.025) = 39.364$	$\chi^2_{24}(0.05) = 36.415$	$\chi^2_{25}(0.025) = 40.646$	$\chi^2_{25}(0.05) = 37.652$
$\chi^2_{24}(0.975) = 12.401$	$\chi^2_{24}(0.95) = 13.848$	$\chi^2_{25}(0.975) = 13.120$	$\chi^2_{25}(0.95) = 14.611$

Scratch Paper

Name: ______________ Student ID: __________________