Beijing-Dublin International College

SEMESTER I RESIT EXAMINATION - 2016/2017

Beijing-Dublin International College

MODULE CODE：BDIC2005J/BDIC1033J
MODULE TITLE：Probability and Statistics

MODULE COORDINATOR NAME*
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Time Allowed: 90 minutes

Instructions for Candidates

All questions carry equal marks. The distribution of marks in the right margin shown as a percentage gives an approximate indication of the relative importance of each part of the question.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted.
No rough-work paper is to be provided for candidates.

Obtained score

Question 1:
Vacancy (Each blank 3 marks)

(1) There are two events A, B. Let $P(A)=0.7$, $P(B)=0.4$, $P(A\overline{B}) = 0.5$, then $P(A\mid B)=$__________.

(2) Suppose the discrete random variable X takes values -3, 0 and 1. It is also known that $P(X=-3)=0.2, P(X=0)=0.3.$ Thus $Var(X)=$_______ .

(3) Suppose the random variable $X \sim B(n,p)$, $E(X)=2.4$, $Var(X)=1.44$, then $n=$____, $p=$_____ .

(4) Let $X_1, X_2, \cdots, X_n (n>2)$ be a sample from $N(\mu, 1)$. Let

$$\overline{X}=\frac{1}{n}\sum_{i=1}^{n} X_i, \quad S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\overline{X})^2,$$

then $\sqrt{n}(\overline{X}-\mu) \sim$_________, $E(S^2)=$_______ .

(5) Let $X_1, \cdots, X_{10}$ be the sample from a normal population $X \sim N(\mu, \sigma^2)$, with $\overline{X}=6$ and $S^2=2.56$, thus the 95 percent confidence interval estimate of μ is

[__________, __________], and the 95 percent confidence interval estimate of σ^2 is

[__________, __________].

The *t* distribution table and the χ^2 distribution table

$t_9(0.025)=2.2622$	$t_9(0.05)=1.8331$	$t_{10}(0.025)=2.2281$	$t_{10}(0.05)=1.8125$
$\chi_9^2(0.025)=19.023$	$\chi_9^2(0.05)=16.919$	$\chi_9^2(0.975)=2.700$	$\chi_9^2(0.95)=3.325$
$\chi_{10}^2(0.025)=20.483$	$\chi_{10}^2(0.05)=18.307$	$\chi_{10}^2(0.975)=3.247$	$\chi_{10}^2(0.95)=3.940$

Obtained score

Part 2: Calculation (14 marks each; Do show all your answer in detail)

(1) A new test has been devised for detecting a particular type of cancer. If the test is applied to a person who has this type of cancer, the probability that the person will have a positive reaction is 0.95 and the probability that the person will have a negative reaction is 0.05. If the test is applied to a person who does not have this type of cancer, the probability that the person will have a positive reaction is 0.05 and the probability that the person will have a negative reaction is 0.95. Suppose that in the general population, one person out of every 1,000 people has this type of cancer. A person is selected at random and the test is implemented,

(a) What is the probability that the person has a positive reaction?

(b) What is the probability that the person has this type of cancer under a positive reaction?

Obtained score

(2) Suppose that the probability density function of the continuous random variable X is

$$f(x) = \begin{cases} 2(x-1), & x \in [1\,,\,2] \\ 0, & elsewhere. \end{cases}$$

Let $Y = 3X$.

Find: (a) $f_Y(y)$, the probability density function of Y; (b) $P(1 < Y < e^2)$; (c) E(Y).

Obtained score

(3) Suppose that the joint probability density of two random variables X and Y are given by

$$f(x,y) = \begin{cases} cx^2, & 0 \le y \le 1 - x^2 \\ 0, & elsewhere. \end{cases}$$

Find:

(a) the constant c and the marginal densities $f_X(x)$, $f_Y(y)$;

(b) whether the two random variables X and Y are independent or not (Please give your reason);

(c) E(X), E(Y).

Obtained score

(4) Suppose $X_1, X_2 \cdots X_n$ is a sample from X, and the probability density function of X is

$$f(x;\theta) = \begin{cases} \theta\, e^{-\theta x}, & x \ge 0, \\ 0, & x < 0, \end{cases}$$

where $\theta > 0$, Find:

(a) the moment estimator $\hat{\theta}$ of θ ;

(b) the maximum likelihood estimator θ^* of θ ;

(c) whether the moment estimator $\hat{\eta} = 1/\hat{\theta}$ of parameter $\eta = 1/\theta$ is unbiased or not (Please give your reason).

Obtained score

(5) Suppose the grade of subject Probability and Statistics is distributed as $N(\mu, \sigma^2)$. A sample of 25 students are selected and their grades are calculated with mean 76.5, standard deviation 9.5. Question: at level of significance 0.05,

(a) can we accept μ= 75 ?

(b) can we accept σ=10 ?

The t distribution table and the χ^2 distribution table

$t_{24}(0.025) = 2.0639$	$t_{24}(0.05) = 1.7109$	$t_{25}(0.025) = 2.0595$	$t_{25}(0.05) = 1.7081$
$\chi^2_{24}(0.025) = 39.364$	$\chi^2_{24}(0.05) = 36.415$	$\chi^2_{25}(0.025) = 40.646$	$\chi^2_{25}(0.05) = 37.652$
$\chi^2_{24}(0.975) = 12.401$	$\chi^2_{24}(0.95) = 13.848$	$\chi^2_{25}(0.975) = 13.120$	$\chi^2_{25}(0.95) = 14.611$

Appendix :