Semester One of Academic Year (2014---2015) of BJUT

《 Probability and Statistics 》 Exam Paper A

Exam Instructions: __Answer ALL Questions__

Honesty Pledge:

I have read and clearly understand the Examination Rules of Beijing University of Technology and University College Dublin and am aware of the Punishment for Violating the Rules of Beijing University of Technology and University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I would accept the punishment thereof.

Pledger: ___________ **Class No:** ___________

BJUT Student ID: ___________ **UCD Student ID**___________

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Notes：

The exam paper has __2__ parts on __8__ pages, with a full score of 100 points. You are required to use the given Examination Book only.

Total Score of the Exam Paper（For teachers' use only）

Item	**Part 1**	**Part 2** (1)	**Part 2** (2)	**Part 2** (3)	**Part 2** (4)	**Part 2** (5)	**Total Score**
Full Score	30	14	14	14	14	14	
Obtained Score							

Obtained score

Part 1: Vacancy (Each blank 3 marks)

(1) There are two events A, B. $P(A)=0.4, P(A \cup B)=0.7$. If A, B are mutually independent,

$P(B)=$_______.

(2) Let random variable X follow a normal distribution $N(\mu,\sigma^2)$, $E(X)=$__________,

$D(X)=$__________.

(3) Let $X_1, X_2, \cdots, X_n (n>2)$ be a sample from $N(\mu,\sigma^2)$.

$$\overline{X}=\frac{1}{n}\sum_{i=1}^{n} X_i, \quad S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\overline{X})^2 .$$

$\sqrt{n}(\overline{X}-\mu)/\sigma \sim$__________, $\sqrt{n}(\overline{X}-\mu)/\sqrt{S^2} \sim$__________, $(n-1)S^2/\sigma^2 \sim$__________.

(4) Let $X_1, \cdots, X_n$ be a sample from $N(\mu,\sigma^2)$. μ and σ^2 are unknown parameters.

The confidence interval of μ with confidence level 95% is [__________ , __________].

The confidence interval of σ^2 with confidence level 95% is [__________ , __________].

Obtained score

Part 2: Calculation (Show all your answer in detail)

(1) Three people *A*, *B* and *C* practice shooting at a target. The probability that *A*, *B* and *C* hit the target is 0.9, 0.8 and 0.6, respectively. Now select any one of them at random,

(a) what is the probability that the person selected hits the target?

(b) If the person actually hit target, what is the probability that the person selected is *A*?

Obtained score

(2) Suppose that the distribution function of the continuous random variable X is

$$F(x)=\begin{cases} a-e^{-0.5x^2}, & x\geq 0 \\ 0, & x<0, \end{cases}$$

Find:

(a) the value of constant a; (b) the probability density function of X $f(x)$.

Obtained score

(3) Suppose that the joint probability density of two random variables *X* and *Y* are given by

$$f(x,y)=\begin{cases} cy^2, & 0\le y\le x\le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find: (a) the value of constant *c*; (b) the marginal densities $f_X(x)$, $f_Y(y)$;

(c) whether the two random variables *X* and *Y* are independent or not.(Please give your reason)

Obtained score

(4) Suppose $X_1, X_2 \cdots X_n$ is a sample from *X*, the probability density function is

$$f_X(x) = \begin{cases} (\alpha+1)x^{\alpha}, & 0 < x < 1 \\ 0, & \text{elsewhere}. \end{cases}$$

where $\alpha > -1$. Find:

(a) the moment estimator of α ;

(b) the maximum likelihood estimator of α.

Obtained score

(5) The mean breaking strength of a certain type of cord has been established from considerable experience at 18.3 ounces. A new machine is purchased to manufacture this type of cord. A sample of 25 pieces obtained from the new machine shows a mean breaking strength of 17.0 ounces, a variance breaking strength of 2.0 ounces. The breaking strength of a certain type of cord is assumed to follow normal distribution $N(\mu, \sigma^2)$.

Question: at level of significance 0.05,

(a) test $H_0: \mu = 18.3 \leftrightarrow H_1: \mu \neq 18.3$, (b) test $H_0: \sigma = 1.5 \leftrightarrow H_1: \sigma \neq 1.5$.

The *t* distribution table and the χ^2 distribution table

$t_{24}(0.025) = 2.0639$	$t_{24}(0.05) = 1.7109$	$t_{25}(0.025) = 2.0595$	$t_{25}(0.05) = 1.7081$
$\chi^2_{24}(0.025) = 39.364$	$\chi^2_{24}(0.05) = 36.415$	$\chi^2_{25}(0.025) = 40.646$	$\chi^2_{25}(0.05) = 37.652$
$\chi^2_{24}(0.975) = 12.401$	$\chi^2_{24}(0.95) = 13.848$	$\chi^2_{25}(0.975) = 13.120$	$\chi^2_{25}(0.95) = 14.611$

Scratch Paper

Name: ______________　　　　Student ID: __________________