Semester One of Academic Year (2015---2016) of BJUT

《 Probability and Statistics》

Module Code: BDIC2005J

Resit Exam Paper

Exam Instructions： Answer ALL Questions

Honesty Pledge：

I have read and clearly understand the Examination Rules of Beijing University of Technology and University College Dublin and am aware of the Punishment for Violating the Rules of Beijing University of Technology and University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I would accept the punishment thereof.

Pledger： ___________   **Class No：** ___________

BJUT Student ID： ___________   **UCD Student ID**___________

Notes：

The exam paper has 2 parts on 4 pages, with a full score of 100 points. You are required to use the given Examination Book only.

Instructions for Candidates

Full marks will be awarded for complete answer to **All** questions.

Instructions for Invigilators

Candidates are allowed to use non-programmable calculators during this examination.

Obtained score

Part 1: Vacancy (Each blank 3 marks)

(1) There are two events A, B. Let $P(A) = 0.7$, $P(B) = 0.4$, $P(A\bar{B}) = 0.6$, then $P(A \mid B) =$ __________.

(2) Suppose the discrete random variable X takes values -3, 0 and 1. It is also known that $P(X = -3) = 0.2$, $P(X = 0) = 0.3$, $P(X = 1) = 0.5$. Thus $E(X^2) =$ ________, and $Var(X) =$ ________.

(3) Suppose the random variable $X \sim B(n, p)$, $E(X) = 2.4$, $Var(X) = 1.44$, then $n =$ _____, $p =$ ______.

(4) Let $X_1, X_2, \cdots, X_n (n > 2)$ be a sample from $N(\mu, \sigma^2)$.

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 .$$

$\bar{X} \sim$ _________, $\sqrt{n}(\bar{X} - \mu)/\sqrt{S^2} \sim$ _________, $E(S^2) =$ ________.

(5) Let $X_1, \cdots, X_{10}$ be the sample from a normal population $X \sim N(\mu, \sigma^2)$, with $\bar{X} = 5$ and $S^2 = 1.96$, Thus a 95 percent confidence interval estimate of μ is [__________, __________].

The t distribution table and the χ^2 distribution table

$t_9(0.025) = 2.2622$	$t_9(0.05) = 1.8331$	$t_{10}(0.025) = 2.2281$	$t_{10}(0.05) = 1.8125$
$\chi_9^2(0.025) = 19.023$	$\chi_9^2(0.05) = 16.919$	$\chi_9^2(0.975) = 2.700$	$\chi_9^2(0.95) = 3.325$
$\chi_{10}^2(0.025) = 20.483$	$\chi_{10}^2(0.05) = 18.307$	$\chi_{10}^2(0.975) = 3.247$	$\chi_{10}^2(0.95) = 3.940$

Obtained score

Part 2: Calculation (Show all your answer in detail)

(1) You ask your neighbor to water a sickly plant while you are on vacation. Without water it will die with probability 0.9; with water it will die with probability 0.2. You are 90 percent certain that your neighbor will remember to water the plant.

(a) What is the probability that the plant will be alive when you return?

(b) If it is dead, what is the probability your neighbor forgot to water it?

(2) Suppose that the probability density function of the continuous random variable *X* is

$$f(x) = \begin{cases} 2(x-1), & x \in [1\ ,\ 2] \\ 0, & elsewhere. \end{cases}$$

Let $Y = e^X$.

Find: (a) the probability density function of *Y* $f_Y(y)$; (b) $P(1 < Y < e^{1.5})$; (c) E(*Y*).

(3) Suppose that the joint probability density of two random variables *X* and *Y* are given by

$$f(x,y)=\begin{cases} c, & -1\le x\le 1,\ x+y\le 1,\ 0\le y\le x+1 \\ 0, & elsewhere. \end{cases}$$

Find: (a) the value of constant *c*; (b) the marginal densities $f_X(x)$, $f_Y(y)$;

(c) whether the two random variables *X* and *Y* are independent or not.(Please give your reason).

(4) Suppose $X_1, X_2 \cdots X_n$ is a sample from *X*, the probability density function is

$$f(x,\theta)=\begin{cases}\dfrac{\theta}{x^{\theta+1}}, & x>1\\ 0, & x\le 1\end{cases}$$

where $\theta > 1$. Find:

(a) the moment estimator of θ ;

(b) the maximum likelihood estimator of θ.

(5) The life of a batch of bulbs is to be studied. Suppose the life of a bulb X (in hours) is normal distributed as $N(\mu, \sigma^2)$. A sample of 10 bulbs are drawn, from which we know the sample mean and sample variance are $\overline{X} = 153,\ S^2 = 64$.

Question: at level of significance 0.05,

(a) test $H_0 : \mu = 150 \leftrightarrow H_1 : \mu \neq 150$ (b) test $H_0 : \sigma = 8 \leftrightarrow H_1 : \sigma \neq 8$