Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION - 2018/2019

School of INSERT SCHOOL: Beijing University of Technology

MODULE CODE and MODULE TITLE

BDIC2005J/BDIC1033J，Probability and Statistics

HEAD OF SCHOOL NAME: BDIC

MODULE COORDINATOR NAME*: Han Min, Zhao Xu, Min Hui

OTHER EXAMINER NAME

Time Allowed: 90 minutes

Instructions for Candidates

All questions carry equal marks. The distribution of marks in the right margin shown as a percentage gives an approximate indication of the relative importance of each part of the question.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

Obtained score

Question 1:

Vacancy (Each blank 3 marks)

$P(AB)=P(A)\cdot P(B)$

$P(A)+P(B)-P(A)P(B)=0.4$

$0.1+P(B)-0.1P(B)$

$P(B)=$

(1) There are two events A and B. P(A)=0.1, P($A\cup B$)=0.4.

If A and B are mutually exclusive, P(B)= 0.3 .

If A and B are mutually independent, P(B)= $\frac{1}{3}$.

(2) Suppose the random variable X~U(a,b). E(X)=5, D(X)=3, a= 2 , b= 8 .

(3) Let $X_1, X_2, \cdots, X_n\ (n>2)$ be a sample from $N(\mu,\sigma^2)$.

$$\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i,\quad S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\overline{X})^2.$$

$\sqrt{n}(\overline{X}-\mu)/\sqrt{S^2}\sim$ t_{n-1} , $(n-1)S^2/\sigma^2\sim$ χ^2_{n-1} .

(4) Let $X_1,\cdots,X_{10}$ be a sample from $N(\mu,\sigma^2)$. We can calculate the sample mean 5 and sample variance 0.25 are respectively. μ and σ^2 are unknown parameters. The confidence interval of μ with confidence level 95% is [4.82 , 5.36]. The confidence interval of σ^2 with confidence level 95% is [0.118 , 0.833].

μ: $\frac{\bar{x}-\mu}{S/\sqrt{n}}\sim t_{n-1}$: $\bar{x}\mp\frac{S}{\sqrt{n}}t_{n-1}\frac{\alpha}{2}=5-\frac{0.5}{\sqrt{10}}\cdot t_9\frac{0.05}{2}=4.64/5.36$

σ^2: $\frac{(n-1)S^2}{\sigma^2}\sim\chi^2_{n-1}$: $\frac{(n-1)S^2}{\chi^2_{n-1}(\frac{\alpha}{2})}=\frac{9\times0.25}{19.023}=0.118$

Obtained score

Question 2: (14 marks)

5 of the 8 guns have been calibrated, 3 have not been calibrated. The probability of a shooter hitting a target with a calibrated gun is 0.8; The probability of the shooter hitting target using an uncalibrated gun is 0.3.

(a) Find the probability of the shooter not hitting target;

(b) Now take one of the 8 guns and shoot, the result is that the shooter hits the target. Find the probability that the gun used to shoot is a calibrated gun.

(a) A: get calibrated gun B: hit

$P(\bar{B})=P(\bar{B}|A)+P(\bar{B}|\bar{A})=\frac{5}{8}\times0.2+\frac{3}{8}\times0.7=0.3875$

(b) $P(A|B)=\frac{P(A)\cdot P(B|A)}{\frac{5}{8}\times0.8+\frac{3}{8}\times0.3}=\frac{\frac{5}{8}\times0.8}{\frac{5}{8}\times0.8+\frac{3}{8}\times0.3}\approx0.8163$

Obtained score

Question 3: (14 marks)

Suppose that the probability density function of the continuous random variable X is

$$f(x)=\begin{cases}\frac{1}{c^2+x^2}, & x\in(0,\frac{\pi}{4});\\ 0, & \text{elsewhere.}\end{cases}$$

Find:

(a) The value of constant c;

(b) P $(\frac{\sqrt{3}\pi}{12}<X\le\frac{\pi}{3})$;

(c) E(X).

(a) $F(1)=\int_0^{\frac{\pi}{4}}f(x)=1 \Rightarrow \frac{1}{c}\arctan\frac{\pi}{4c}=1$, $c=\frac{\pi}{4}$

$F(x)=\frac{\pi}{4}\arctan\frac{4}{\pi}x$

$P(\frac{\sqrt{3}\pi}{12}<x\le\frac{\pi}{3})=F(\frac{\pi}{3})-F(\frac{\sqrt{3}\pi}{12})=\frac{\pi}{4}(\arctan\frac{4}{3}-\arctan\frac{\sqrt{3}}{3})$

$=18.17$

(c) $E(x)=\int_0^{x}xf(x)dx=\int_0^{\frac{\pi}{4}}\frac{x}{c^2+x^2}dx=\frac{1}{2}\ln\frac{x^2+c^2}{c^2}=\frac{1}{2}\ln\frac{x^2}{c^2}$

Obtained score

Question 4: (14 marks)

Suppose that the joint probability density of two random variables X and Y are given by

$$f(x,y)=\begin{cases} c\cdot y^3(2-x), & 0\le x\le 1, 0\le y\le x, \\ 0, & elsewhere. \end{cases}$$

Find:

(a) The value of constant c;

(b) The marginal densities $f_X(x),\ f_Y(y)$;

(c) Whether the two random variables X and Y are independent or not? (Please give your reason);

(d) E(XY)

Obtained score

Question 5: (14 marks)

Suppose $X_1, X_2 \cdots X_n$ is a sample from X, and the probability density function of X is

$$f(x;\theta)=\begin{cases} \theta^2 x\, e^{-\theta x}, & x>0, \\ 0, & x\le 0, \end{cases}$$

where $\theta>0$. Find:

(a) The moment estimator of θ;

(b) The maximum likelihood estimator of θ.

Obtained score

Question 6: (14 marks)

The capacities (in ampere-hours) of 10 batteries were recorded as follows:

102, 97, 101, 103, 101, 98, 99, 104, 103, 98

Under the assumption that the capacity is normal distributed as $N(\mu,\sigma^2)$.

Question: at level of significance 0.05,

(a) test $H_0:\mu=100 \leftrightarrow H_1:\mu\ne 100$, (b) test $H_0:\sigma^2=2.5 \leftrightarrow H_1:\sigma^2>2.5$.

The t distribution table and the χ^2 distribution table

$t_9(0.025)=2.2622$	$t_9(0.05)=1.8331$	$t_{10}(0.025)=2.2281$	$t_{10}(0.05)=1.8125$
$\chi_9^2(0.025)=19.023$	$\chi_9^2(0.05)=16.919$	$\chi_9^2(0.975)=2.700$	$\chi_9^2(0.95)=3.325$
$\chi_{10}^2(0.025)=20.483$	$\chi_{10}^2(0.05)=18.307$	$\chi_{10}^2(0.975)=3.247$	$\chi_{10}^2(0.95)=3.940$

Appendix :