Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION - 2017/2018

BDIC2005J / BDIC1033J

Probability and Statistics

HEAD OF SCHOOL WU Wenying

MODULE COORDINATOR LIU Ling/ZHAO Xu

Time Allowed: 90 minutes

Instructions for Candidates

All questions carry equal marks. The distribution of marks in the right margin shown as a percentage gives an approximate indication of the relative importance of each part of the question.

BJUT Student ID: ________________ **UCD Student ID: ________________**

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: ____ (Signature)

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

Obtained score

Question 1:
Vacancy (Each blank 3 marks)

(1) There are two events A and B, P(A)=0.6，P(B)=0.3. If A and B are mutually independent,

P(A∪B) = 0.9, P(A-B)= 0.6.

$e^{-\lambda}\frac{\lambda^k}{k!} = e^{-\lambda} = \frac{1}{2}$

$-\lambda = \ln\frac{1}{2}$

(2) $X \sim P(\lambda)$, P(X=0) =1/2, λ = $\ln 2$.

(3) Let $X_1, X_2, \cdots, X_n (n > 2)$ be a sample from $N(\mu, \sigma^2)$.

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 \cdot$$

$\sqrt{n}(\overline{X} - \mu)/\sigma \sim$ $N(0,1)$, $\sqrt{n}(\overline{X} - \mu)/\sqrt{S^2} \sim$ t_{n-1},

$(n-1)S^2/\sigma^2 \sim$ χ^2_{n-1}.

(4) Let $X_1, \cdots, X_n$ be a sample from $N(\mu, \sigma^2)$. μ and σ^2 are unknown parameters.

The confidence interval of μ with confidence level 95% is
[$\bar{x} - \frac{s}{\sqrt{n}} t_{n-1}(0.025)$, $\bar{x} + \frac{s}{\sqrt{n}} t_{n-1}(0.025)$].

$\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_{n-1}$

The confidence interval of σ^2 with confidence level 95% is
[$\frac{(n-1)s^2}{\chi^2_{n-1}(0.975)}$, $\frac{(n-1)s^2}{\chi^2_{n-1}(0.025)}$].

$\mu: \bar{x} \mp \frac{s}{\sqrt{n}} t_{n-1}(\frac{\alpha}{2})$

Obtained score

Question 2: (14 marks)

A:2 B:3 C:5

In our warehouse, there are 10 boxes of products provided by three factories. Exactly 2 boxes are provided by factory A; 3 boxes are provided by factory B; the others are provided by factory C. And we also know that the qualification rates for the three factories are 95%, 90% and 96% separately.

Find:
(a) The qualification rate of the products in our warehouse;
(b) Now we randomly take a product from a randomly chosen box. And we can see the selected product is qualified. What is the probability that it is provided by factory A? What is the probability that it is provided by factory B? What is the probability that it is provided by factory C?

(a) N is qualification

$P(N) = P(N|A) + P(N|B) + P(N|C) = \frac{2}{10}\times 95\% + \frac{3}{10}\times 90\% + \frac{5}{10}\times 96\% \approx 0.94$

(b) $P(A|N) = \frac{P(A)\cdot P(N|A)}{P(A)\cdot P(N|A) + P(B)P(N|B) + P(C)\cdot P(N|C)} = \frac{0.19}{0.94} = 0.201$

Obtained score

Question 3: (14 marks)
Suppose, for a town, the electricity consumption per day X is a continuous random variable. And the density function of X is

$P(B|N) = 0.287$ $P(C|N) = 0.554$

$$f(x) = \begin{cases} cx(1-x)^2, 0 < x < 1 \\ 0, else \end{cases}$$

(a) $F(x) = 1$, $x \to \infty$ $\int_0^1 cx(1-x)^2 dx = 1$ $c = 12$

Find:
(a) The value of constant c;
(b) P(0.5<X<1.3);
(c) E(X).

$P(0.5 < x < 1.3) = \int_{0.5}^{1.3} f(x) = \int_{0.5}^{1} 12x(1-x)^2 dx = 0.3125$

$E(x) = \int_0^1 xf(x) = \int_0^1 12x^2(1-x)^2 dx = 12\int_0^1 (x^2 - 2x^3 + x^4) dx$

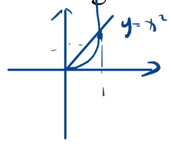
$= 12(\frac{1}{3}x^3 - \frac{2}{4}x^4 + \frac{1}{5}x^5)\Big|_0^1 = 12(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}) = 0.4$

Obtained score

Question 4: (14 marks)

Suppose that the joint probability density of two random variables X and Y are given by

$$f(x,y)=\begin{cases} c, x^2 \le y \le x \\ 0, else \end{cases}$$



Find:

(a) The value of constant c;

$(a)\ \int_0^1 \int_{x^2}^{x} c\, dx dy = 1$

$\int_0^1 c(x-x^2)\, dx = 1$

$c\left(\frac{1}{2}x^2-\frac{1}{3}x^3\right)\Big|_0^1 = 1 \qquad c=6$

(b) The marginal densities $f_X(x),\ f_Y(y)$;

(c) Whether the two random variables X and Y are independent or not? (Please give your reason).

$(b)\ f_X(x) = \int_{x^2}^{x} 6\, dy = 6(x-x^2)$

$f_Y(y) = \int_y^{\sqrt{y}} 6\, dx = 6(\sqrt{y}-y)$

$f(x,y) \ne f_X(x)\cdot f_Y(y)$ so in not independent

(d) E(*XY*)

$d\ E(XY) = \int_0^1 \int_{x^2}^{x} 6xy\, dx dy$

$= \int_0^1 6x \cdot \frac{1}{2}(x^2-x^4)\, dx = \frac{1}{4}$

Obtained score

Question 5: (14 marks)

Suppose $X_1, X_2, \ldots, X_n$ is a sample from X, and the probability density function of X is

$$f(x)=\begin{cases} \frac{1}{\theta}e^{-\frac{x}{\theta}}, x \ge 0 \\ 0, else \end{cases}$$

$(a)\ E(x) = \int_0^{\infty} x f(x) = \int_0^{\infty} \frac{x}{\theta} e^{-\frac{x}{\theta}}\, dx$

$= -\int_0^{\infty} x\, de^{-\frac{x}{\theta}} = -\left(xe^{-\frac{x}{\theta}} - \int_0^{\infty} e^{-\frac{x}{\theta}}\, dx\right)$

$= -\theta e^{-\frac{x}{\theta}}\Big|_0^{\infty} = \theta$

Find: $E(x) = \bar{x} \quad \hat{\theta} = \bar{x} \qquad E(\hat{\theta}) = E(\bar{x}) = \theta$ so it is unbiased

(a) The moment estimator $\hat{\theta}$ of θ, is it unbiased?;

(b) The maximum likelihood estimator θ^* of θ.

$(b)\ L(\theta) = \frac{1}{\theta^n} \cdot e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i}$

$\ln L(\theta) = n \ln\frac{1}{\theta} - \frac{1}{\theta}\sum_{i=1}^{n} x_i$

$\frac{\partial \ln L(\theta)}{\partial(\theta)} = -\frac{n}{\theta} + \frac{\sum_{i=1}^{n} x_i}{\theta^2} = 0$

$\theta^* = \bar{x}$

Obtained score

Question 6: (14 marks)

Suppose the nicotine content of a brand of cigarette follows a normal distribution. Now we randomly take 10 cigarettes, the average nicotine content $\bar{X}$=18.6 milligram, the standard deviation S=2.4 milligram. Take the significant level α=0.05.

(a) Can we accept μ=18?

(b) Can we accept σ= 2?

The t distribution table and the χ^2 distribution table

$t_9(0.025)=2.2622$	$t_9(0.05)=1.8331$	$t_{10}(0.025)=2.2281$	$t_{10}(0.05)=1.8125$
$\chi_9^2(0.025)=19.023$	$\chi_9^2(0.05)=16.919$	$\chi_9^2(0.975)=2.700$	$\chi_9^2(0.95)=3.325$
$\chi_{10}^2(0.025)=20.483$	$\chi_{10}^2(0.05)=18.307$	$\chi_{10}^2(0.975)=3.247$	$\chi_{10}^2(0.95)=3.940$

$a:\quad H_0: \mu=18 \qquad H_1: \mu \ne 18$

$a: \frac{\bar{x}-\mu}{s/\sqrt{n}} \sim t_{n-1}$

$P\left(-t_{n-1}\frac{\alpha}{2} \le \frac{\bar{x}-\mu}{s/\sqrt{n}} \le t_{n-1}\frac{\alpha}{2}\right) = 1-\alpha$

reject domain: $\frac{|\bar{x}-\mu_0|}{s/\sqrt{n}} \ge t_{n-1}\left(\frac{\alpha}{2}\right)$

$\frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{18.6-18}{2.4/\sqrt{10}} = 0.790$

$t_9\ 0.025 = 2.2622 > 0.790$ so Acceptable

Appendix :

6) $H_0: \sigma^2 = 4 \quad H_1: \sigma^2 \neq 4$

$Q = \frac{(n-1)s^2}{\sigma^2} \sim \chi_{n-1}$

$P\left[\frac{(n-1)s^2}{\sigma_0^2} \le c_1 \text{ or } \frac{(n-1)s^2}{\sigma_0^2} \ge c_2\right] = 0.05$

Reject region: $\left[\frac{(n-1)s^2}{\sigma_0^2} \le \chi^2_{n-1}(1-\frac{\alpha}{2})\right] \cup \left[\frac{(n-1)s^2}{\sigma_0^2} \ge \chi^2_{n-1}(\frac{\alpha}{2})\right]$

$\frac{(n-1)s^2}{\sigma^2} = \frac{9 \times 2.4^2}{4} = 12.96$

$\chi_9\ 0.975 = 2.7 \quad \chi_9\ 0.025 = 19.023$

so $\frac{(n-1)s^2}{\sigma^2} \in \left[\chi^2_9(1-\frac{\alpha}{2}),\ \chi^2_9(\frac{\alpha}{2})\right]$

Accept $\sigma = 2$