Beijing-Dublin International College

SEMESTER 2 FINAL EXAMINATION – 2024/2025

School of Mathematics and Statistics

BDIC1031J Maths 3 (Advanced Mathematics)

HEAD OF SCHOOL: Wenying WU

MODULE COORDINATOR: Yuehong FENG

Time Allowed: 95 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.

No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.
Time allowed is **95** minutes.
The exam paper has **2** sections on **6** pages, with a full score of 100 marks.
You are required to use only the provided **Examination Book** for answers.

SECTION A — Brief Answer Questions

This section is worth a total of **60** marks, with each question worth **5** marks.

1. Compute the sum function

$$\sum_{n=1}^{\infty} nx^{n-1} = \_\_\_\_\_.$$

2. Compute the sum function

$$\sum_{n=1}^{\infty} (2n+1)x^n = \_\_\_\_\_.$$

3. Compute the sum function

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = \_\_\_\_\_.$$

4. Compute

$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots = \_\_\_\_\_.$$

5. The directional derivative of function $z = x^2 + y^2$ at point $(1,\ 2)$ in the direction from point $(1,\ 2)$ to point $(2,\ 2+\sqrt{3})$ is _____.

6. Given $f(x, y) = \arctan\dfrac{x+y}{1-xy}$ find the second partial derivative

$$\frac{\partial^2 f}{\partial x^2} = \_\_\_\_\_.$$

7. The angle of inclination between the tangent line of the curve $\begin{cases} z = \dfrac{1}{4}(x^2 + y^2), \\ y = 4, \end{cases}$ at the point $(2, 4, 5)$ and the positive direction of x-axis is _____

8. Let $z = e^{x-2y}$, and $x = \sin t,\ y = t^3$. Find $\dfrac{dz}{dt} =$ _____

9. Let $\sqrt{x^2+y^2} = \arctan\frac{y}{x}$.

   Find $\frac{dy}{dx} =$ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

10. Let $\frac{x}{z} = \ln(zy)$.

    Find $\frac{\partial z}{\partial x} =$ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ and $\frac{\partial z}{\partial y} =$ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

11. The equations of tangent line and normal plane of the curve $C: \begin{cases} x = t - \sin t, \\ y = 1 - \cos t, \\ z = 4\sin\frac{t}{2}. \end{cases}$ at the point $(x(t_0), y(t_0))$, where $t_0 = \frac{\pi}{2}$ are respectively

    _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ and _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

12. Let σ_1 be a tangent plane of the surface $x^2 + 2y^2 + z^2 = 1$. Let σ_2 be another plane $x - y + 2z = 0$. If σ_1 is perpendicular to σ_2, then the equation of σ_1 is

    _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

SECTION B — Extended Answer Questions

Write your answers on the **Examination Book** provided.
This section is worth a total of **40** marks. The marks of each question are as shown.

13. (5 marks) Consider a function $f(x,y)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}}, & x^2+y^2\neq 0,\\ 0, & x^2+y^2=0.\end{cases}$

Show that the function $f(x,y)$ must be partial derivable at (0, 0), but it is not differentiable at this point.

14. (5 marks) Show that the limit $\lim\limits_{(x,y)\to(0,0)}\dfrac{xy}{x+y}$ does not exist.

15. (6 marks) For what value of p does the series $\sum_{n=2}^{\infty} \frac{1}{n^p \ln n}$ converge?

16. (6 marks) Prove that the function

$$f(x,y)=\begin{cases}(x^2+y^2)\sin\dfrac{1}{x^2+y^2}, & x^2+y^2\neq 0,\\ 0, & x^2+y^2=0,\end{cases}$$

is differentiable at the origin $(0,\ 0)$, but the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ are both discontinuous at the point $(0,\ 0)$.

17. (9 marks) Compute the radius of convergence and interval of convergence for the following power series.

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} x^{2n}$$

18. (9 marks) Show that the lines $L_1 : \frac{x-1}{1} = \frac{y+2}{3} = \frac{z-4}{-1}$ and $L_2 : \frac{x}{2} = \frac{y-3}{1} = \frac{z+3}{4}$ are skew, which means that the two lines do not intersect and are not parallel, therefore they do not lie in the same plane. Find the distance between them.

Glossary

Angle of inclination	夹角
Curve	曲线
Differentiable	可微
Directional derivative	方向导数
Normal plane	法平面
Partial derivable	可偏导
Perpendicular	垂直
Positive direction of x-axis	x 轴正向
Tangent line	切线
Radius of convergence	收敛半径
Tangent plane	切平面
Surface	曲面
Sum function	和函数