Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION – 2019/2020

School of Mathematics and Statistics
BDIC1029J & BDIC1025J Maths 1 (Advanced Mathematics)

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MODULE COORDINATOR: Yanru PING
OTHER EXAMINER: Yuehong FENG
OTHER EXAMINER: Bin ZHENG

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **4** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

Section A: Fill-in-the-blank Questions

This section is worth a total of **80** marks, with each question worth **5** marks.

1. Evaluate

$$\lim_{x\to\infty} x \cdot \sin\frac{1}{x} = \_\_\_\_\_$$

2. Find the limit

$$\lim_{x\to\infty} \frac{x^2 - x + 1}{1 - x^2 - x^3} \cdot \cos x = \_\_\_\_\_$$

3. Find the limit

$$\lim_{x\to\frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}} = \_\_\_\_\_.$$

4. Let $f(x)$ be the function

$$f(x) = \frac{1}{x^2 + 3x + 2}.$$

Find the higher order derivative $f^{(20)}(x) = \_\_\_\_\_$.

5. Let $f(x)$ be the function

$$f(x) = \ln(\sqrt{x^2 + 1} + x).$$

Find the differential $df(x) = \_\_\_\_\_$.

6. Find the limit

$$\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}} = \_\_\_\_\_.$$

7. Given

$$\lim_{x\to 0}\frac{e^{h(x)}-1}{\tan x}=5,$$

find the limit

$$\lim_{x\to 0}\frac{h\,(x)}{x}=\_\_\_\_\_.$$

8. Given $f(x)=\dfrac{\sin x}{x^2+x}$,then the equation of the horizontal asymptote of the curve $f(x)$ is $y=$ _____ whereas the equation of the vertical asymptote of the curve $f(x)$ is $x=$ _____

9. Given $f'(1)=3$ exists,

$$\lim_{x\to 0}\frac{f(1+\sin 2x)-f(1)}{x}=\_\_\_\_\_,$$

.

10. Given

$$y=x\cdot\cos 2x,$$

find the higher order derivative $y^{(20)}=$ _____.

11. Find the limit

$$\lim_{x\to 0}\frac{x}{\sqrt[5]{x+1}-1}=\_\_\_\_\_.$$

12. Given

$$\begin{cases} x=\cos t, \\ y=\sin t, \end{cases} \qquad t \text{ being a parameter, } \ t\in\mathbb{R},$$

evaluate

$$\frac{dy}{dx}=\_\_\_\_\_, \qquad \frac{d^2y}{dx^2}=\_\_\_\_\_.$$

13. Find $\dfrac{dy}{dx}$, where

$$y=(\frac{x}{x+1})^x.$$

14. Given

$$xy + e^y + y = 2$$

then $y' =$_____.

15. Evaluate the values of constants a and b, such that the function

$$f(x) = \begin{cases} 1 + \sin 2x, & x \leq 0 \\ a + bx, & x > 0 \end{cases}$$

is derivable at $x = 0$.

(Hint: Find $a =$ _____, $b =$ _____)

16. Given $f(x) = 2x^3 - 9x^2 + 12x + 1$, the monotonically decreasing interval of $f(x)$ is _____, its local maximum value is _____ and local minimum value is _____ .

Advanced Mathematics (Module 1)

Section B: Extended Answer Questions

This section is worth a total of **20** marks, with each question worth **5** marks.

17. Suppose $f(x)$ is a continuous function over the interval $[0,1]$. $f(x)$ is derivable on the point $(0,1)$, with evaluation $f(1)=0$.

Try to prove that there exists at least one number $\xi \in (0,1)$, such that

$$f'(\xi) = -\frac{3f(\xi)}{\xi}.$$

18. Prove that the identity $2\arctan x + \arcsin\dfrac{2x}{1+x^2} = \pi$, where $x \geq 1$.

19. Determine the limit

$$\lim_{x\to 0} \frac{x\cdot\cos x - \sin x}{x^2 \arctan x}$$

by using equivalent infinitesimal substitution theorem and L'Hospital Law.

20. Show that $\ln(1+x) \geq \dfrac{\arctan x}{1+x}$ for $x \geq 0$.

Glossary

Curve	曲线
Decreasing	递减
Derivative	导数
Differentiable	可微分的
Differential	微分
Discontinuity	不连续
Higher order derivative	高阶导数
Horizontal asymptote	水平渐近线
Vertical asymptote	铅垂渐近线
Infinitesimal	无穷小量
Local maximum	极大值
Logarithmic differentiation	对数求导法
Local Minimum	极小值
Monotonically decreasing	单调递降
Tangent line	切线