Beijing-Dublin International College (BDIC)

FIRST SEMESTER, Academic Year 2016–2017

Campus: Beijing University of Technology (BJUT)

BDIC1029J/BDIC1025J Advanced Mathematics (Module 1) — Final Exam

Honesty Pledge:

I have read and clearly understand the Examination Rules of Beijing University of Technology and University College Dublin and am aware of the Punishment for Violating the Rules of Beijing University of Technology and University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I would accept the punishment thereof.

Pledger: ______________________ Class NO: ______________________

BJUT Student ID: ________________ UCD Student ID: ________________

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **7** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

BDIC1029J/BDIC1025J Advanced Mathematics (Module 1)

Section A: Fill-in-the-blank Questions

This section is worth a total of **80** marks, with each question worth **5** marks.

1. Find the limit

$$\lim_{x\to\frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}} = \_\_\_\_\_\_\_\_\_\_.$$

2. Given

$$\lim_{x\to 0} \frac{f(x)}{\sin x \cdot [3^x - 1]} = 2,$$

find the limit

$$\lim_{x\to 0} \frac{f(x)}{x^2} = \_\_\_\_\_\_\_\_\_\_.$$

3. Find the limit

$$\lim_{x\to 2} \frac{x^{\sin x} - 1}{x \ln x} = \_\_\_\_\_\_\_\_\_\_.$$

4. Find the limit

$$\lim_{x\to 0} \frac{\ln(1 + \sin x)}{x} = \_\_\_\_\_\_\_\_\_\_.$$

5. Find the limit

$$\lim_{x\to 1} \left(\frac{3}{1 - x^3} - \frac{2}{1 - x^2} \right) = \_\_\_\_\_\_\_\_\_\_.$$

6. Find the following limit, where $a > 0$:

$$\lim_{n\to\infty} \sqrt[n]{1 + a^n} = \_\_\_\_\_\_\_\_\_\_.$$

7. Find the limit

$$\lim_{x\to 0}\frac{\tan x-\sin x}{\sqrt{1-x^3}-1}=\_\_\_\_\_\_\_\_\_\_.$$

8. Given

$$\lim_{x\to 1}\frac{x^2+2x-a}{x-1}=b,$$

evaluate $a=$ __________, and $b=$ __________.

9. Let $f(x)$ be a continuous function at the point $x=1$, with

$$\lim_{x\to 1}\frac{f(x)}{x-1}=3.$$

Evaluate $f'(1)=$ __________.

10. Given

$$y=x^2\cdot\cos 6x,$$

find the higher order derivative $y^{(50)}=$ __________.

11. Let $f(x)$ be the function

$$f(x)=\frac{x}{x-1}.$$

Find the higher order derivative $f^{(n)}(x)=$ __________.

12. Consider a curve given by an equation

$$C:\quad y^5+2y-x-3x^7=0.$$

Determine its derivative at point $x=0$, that is, $\frac{dy}{dx}|_{x=0}=$ __________, and then give the equation of the tangent line at the point $x=0$: __________.

13. Given

$$\begin{cases} x = t^2, \\ y = \ln\left(1 + t^2\right), \end{cases} \qquad t \text{ being a parameter}, \quad t \in \mathbb{R}^+,$$

evaluate

$$\frac{dy}{dx} = \underline{\qquad\qquad}, \qquad \frac{d^2y}{dx^2} = \underline{\qquad\qquad}.$$

14. Given

$$y = \sqrt[5]{\frac{x(x-5)(x-1)}{x^2+2}},$$

find

$$\frac{dy}{dx} = \underline{\qquad\qquad}.$$

15. Let $f(x)$ be a continuous function, with $f'(x) = \ln(1 + x^2)$. Supposing

$$y = f\left(\frac{3x-2}{3x+2}\right),$$

we can evaluate

$$\left.\frac{dy}{dx}\right|_{x=0} = \underline{\qquad\qquad}.$$

16. Let $f(x)$ be the function

$$f(x) = \ln(\sin x + \sqrt{1 + \sin^2 x}).$$

Find the differential（微分）$df(x) = \underline{\qquad\qquad}$.

BDIC1029J/BDIC1025J Advanced Mathematics (Module 1)

Section B: Extended Answer Questions

This section is worth a total of **20** marks, with each question worth **5** marks.

17. Let $y\left(x\right)$ be the function

$$y = \frac{2^{\frac{1}{x}} - 1}{2^{\frac{1}{x}} + 1} + \sin\left(x-2\right)\sin\frac{1}{x-2}.$$

Find all the discontinuous point(s) of $y\left(x\right)$, and determine the type of discontinuity for each point.

18. Consider a sequence $\{x_n\}$, $n = 1, 2, \cdots$, defined by a recursive formula

$$x_{n+1} = \sqrt{3 + x_n}, \qquad \text{with} \qquad x_1 = 3.$$

Prove that the limit $\lim\limits_{n\to\infty} x_n$ exists, and find that limit.

* *Hint: Make use of the monotonic sequence theorem* （单调有界数列一定存在极限）.

19. Let C be the curve given by the function

$$y = \frac{(2x+3)\ln(1+x)}{x(x-5)},$$

Find the equation of the vertical asymptote of the curve C (求铅垂渐近线方程).

20. Let $f\left(x\right)$ be a continuous function over the interval $[0,1]$, satisfying $f\left(0\right) = f\left(1\right)$. Prove that there exists at least one number $c \in [0,1]$, such that

$$f\left(c\right) = f\left(c + \frac{1}{3}\right).$$

* *Hint: Make use of the zero point theorem.*

THIS PAGE IS INTENTIONALLY LEFT BLANK.