Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION – 2016/2017

School of Mathematics and Statistics
BDIC1029J & BDIC1025J Maths 1 (Advanced Mathematics)

HEAD OF SCHOOL: Gary McGuire
MODULE COORDINATOR: Yanru Ping
OTHER EXAMINER: Yuehong Feng

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **5** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

Section A: Fill-in-the-blank Questions

This section is worth a total of **80** marks, with each question worth **5** marks.

1. Given

$$\lim_{x\to 0} \frac{\sin x}{e^{ax}-1} = 5,$$

evaluate $a =$ _____.

2. Given

$$\lim_{x\to 0} \frac{\ln\left[1+\frac{f(x)}{\sin x}\right]}{2^x-1} = 3,$$

find the limit

$$\lim_{x\to 0} \frac{f\left(x\right)}{x^2} = \_\_\_\_\_.$$

3. Find the limit

$$\lim_{x\to 0} \frac{e^{\sin x}-1}{x} = \_\_\_\_\_.$$

4. Find the limit

$$\lim_{x\to 0} \frac{\ln\cos x}{x^2} = \_\_\_\_\_.$$

5. Find the limit

$$\lim_{x\to 1}\left(\frac{4}{1-x^4}-\frac{3}{1-x^3}\right)=\_\_\_\_\_.$$

6. Find the limit

$$\lim_{n\to\infty}\sqrt[n]{3^n+1}=\_\_\_\_\_.$$

7. Find the limit

$$\lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{\sqrt{1-x^3}-1}=\_\_\_\_\_$$

8. Given

$$\lim_{x\to 1}\frac{x^2+x+a}{x-1}=b,$$

evaluate $a=$ _____, and $b=$ _____.

9. Let $f(x)$ be a continuous function at the point $x=0$, with

$$\lim_{x\to 0}\frac{f(x)}{x}=5.$$

Evaluate $f'(0)=$ _____.

10. Given

$$y=\cos^2 x,$$

find the higher order derivative $y^{(n)}=$ _____.

11. Let $f(x)$ be the function

$$f(x) = \frac{x^2}{x-1}.$$

Find the higher order derivative $f^{(n)}(x) =$ _____.

12. Consider two curves given by the equations

$$C_1: \quad y = x^2 + ax + b, \qquad\qquad C_2: \quad 2y = -1 + xy^3.$$

Suppose C_1 and C_2 contact at the point $P(1,-1)$, and they have the same tangent line at P. Then a and b can be evaluated as $a =$ _____, $b =$ _____.

13. Given

$$\begin{cases} x = \ln\left(1+t^2\right), \\ y = \arctan t, \end{cases} \qquad t \text{ being a parameter, } \ t \in \mathbb{R},$$

evaluate

$$\frac{dy}{dx} = \_\_\_\_\_, \qquad\qquad \frac{d^2y}{dx^2} = \_\_\_\_\_.$$

14. Given

$$y = \frac{x-5}{\sqrt[3]{x^2+2}},$$

find

$$\frac{dy}{dx} = \_\_\_\_\_.$$

15. Let $f(x)$ be a continuous function, with $f'(x) = \arctan x$. Supposing

$$y = f\left(\frac{3x-2}{3x+2}\right),$$

we can evaluate

$$\left.\frac{dy}{dx}\right|_{x=0} = \_\_\_\_\_.$$

16. Let $f(x)$ be the function

$$f(x) = \tan^3 x.$$

Find the differential $df(x) = \_\_\_\_\_$.

Advanced Mathematics (Module 1)

Section B: Extended Answer Questions

This section is worth a total of **20** marks, with each question worth **5** marks.

17. Let $y(x)$ be the function

$$y = \frac{2^{\frac{1}{x}} - 1}{2^{\frac{1}{x}} + 1} + \sin(x-1)\sin\frac{1}{x-1}.$$

Find all the discontinuous point(s) of $y(x)$, and determine the type of discontinuity for each point.

18. Consider a sequence $\{x_n\}$, $n = 1, 2, \cdots$, defined by a recursive formula

$$x_{n+1} = \sqrt{x_n + 2}, \qquad x_1 = \sqrt{2}.$$

Prove that the limit $\lim_{n\to\infty} x_n$ exists, and find that limit.

19. Let C be the curve given by the function

$$y = \frac{-2x^2 + x - 5}{1 - 5x^2}.$$

Find the equation of the horizontal asymptote of the curve C.

20. Let $f(x)$ be a continuous function over the interval $[0, 1]$, satisfying $f(0) = f(1)$. Prove that there exists at least one number $c \in [0, 1]$, such that

$$f(c) = f\left(c + \frac{1}{3}\right).$$