Beijing-Dublin International College (BDIC)

FIRST SEMESTER, Academic Year 2015–2016

Campus: Beijing University of Technology (BJUT)

BDIC1029J & BDIC1025J, Advanced Mathematics

(Module 1; Engineering and Finance) Final Exam (Supplemental)

Honesty Pledge:

I have read and clearly understand the Examination Rules of Beijing University of Technology and University College Dublin and am aware of the Punishment for Violating the Rules of Beijing University of Technology and University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I would accept the punishment thereof.

Pledger: _____ Class NO: _____

BJUT Student ID: _____ UCD Student ID: _____

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **4** pages, with a full score of 100 marks.

You are required to use only the provided **Examination Book** for answers.

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SECTION A — Gap-filling QUESTIONS

In each question, choose **at most one** option.

Circle the preferred choice on the **Examination Book** provided.

This section is worth a total of **84** marks, with each question worth **6** marks.

1. Given

$$\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5,$$

find $a=$ _____, $b=$ _____ .

2. Evaluate the limit

$$\lim_{x\to 1}\frac{x^5-1}{x^4-1}=\_\_\_\_\_$$

3. Evaluate the limit

$$\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}=\_\_\_\_\_$$

4. Evaluate the limit

$$\lim_{x\to 0}\frac{1-\cos x^2}{x^4}=\_\_\_\_\_$$

5. Evaluate the limit

$$\lim_{x\to 1}\left(\frac{4}{1-x^4}-\frac{3}{1-x^3}\right)=\_\_\_\_\_$$

6. Evaluate the limit

$$\lim_{n\to\infty}\sqrt[n]{1+5^n}=\_\_\_\_\_$$

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7. Evaluate the limit

$$\lim_{x\to 0}\frac{\sin 2x}{\sin 5x}\_\_\_\_\_$$

8. Evaluate the limit

$$\lim_{x\to 0}\frac{x}{\sqrt{1+x}-1} = \_\_\_\_\_$$

9. Given $f(x) = \sin^2 x$, find the 10th order derivative of $f(x)$:

$$f^{(10)}(x) = \_\_\_\_\_$$

10. Evaluate the higher order derivative of $f(x) = \frac{1}{x^2+3x+2}$:

$$f^{(n)} = \_\_\_\_\_$$

11. Given $\begin{cases} x = \sin t, \\ y = \cos t \end{cases}$, find the derivative

$$\frac{dy}{dx} = \_\_\_\_\_$$

12. Given $f(x) = \sqrt{(x+2)(x+3)(x+4)}$, find the derivative of $f(x)$:

$$f'(x) = \_\_\_\_\_.$$

13. Suppose a function $y = y(x)$ is determined by the equation $xy + y^3 - e^x = 5$. Then the derivative of $y = y(x)$ is

$$y' = \_\_\_\_\_.$$

14. Letting $f(x) = \sin^3 x$, the differential of this $f(x)$ is

$$df(x) = \_\_\_\_\_.$$

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SECTION B — EXTENDED ANSWER QUESTIONS

Write your answers on the **Examination Book** provided.

This section is worth a total of **16** marks, with each question worth **8** marks.

15. (**8 marks**) Suppose

$$f(x) = \frac{\sin x}{x} + \frac{\cos(x-1)}{x-1}.$$

Find all the discontinuous points of $f(x)$, and determine the type of discontinuity for each of them:

- if the discontinuity belongs to Type-I, point out whether it is a jump （跳跃间断点）or a removable （可去间断点）discontinuity;
- if it belongs Type-II, point out whether it is an infinite discontinuity （无穷间断点）or a shock discontinuity (震荡间断点).

16. (**8 marks**) Prove that there is at lest one solution（解）for the equation

$$x + \sin x - 5 = 0.$$

** Hint*（提示）*: Try to use the intermediate value theorem for continuous functions*（利用连续函数的介值定理证明）.