Beijing-Dublin International College (BDIC)

FIRST SEMESTER, Academic Year 2014–2015

Campus: Beijing University of Technology (BJUT)

Advanced Mathematics (Module 1) — Final Exam (Supplemental)

Honesty Pledge:

I have read and clearly understand the Examination Rules of Beijing University of Technology and University College Dublin and am aware of the Punishment for Violating the Rules of Beijing University of Technology and University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I would accept the punishment thereof.

Pledger: _____ Class NO: _____

BJUT Student ID: _____ UCD Student ID: _____

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **4** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

Advanced Mathematics (Module 1)

Section A: Fill-in-the-blank Questions

This section is worth a total of **85** marks, with each question worth **5** marks.

1. Find the equation of the vertical asymptote of the function

$$f(x) = \frac{x \sin(x-1)}{x^2 - 3x + 2}$$

Your answer: _____.

2. The discontinuous point of the function

$$f(x) = \frac{2^x + 1}{2^x - 1}$$

is at _____, which is _____ (select one answer from: (A) an infinite discontinuity, (B) a jump discontinuity, or (C) a removable continuity).

3. Given a function

$$f(x) = \frac{x}{1 + 3x - x^2}$$

find the equation of its horizontal asymptote: _____.

4. Evaluate the limit:

$$\lim_{n \to \infty} \sqrt[n]{3^n + 2^n} = \_\_\_\_\_$$

5. Evaluate the limit:

$$\lim_{x \to 0} x \sin \frac{1}{x} = \_\_\_\_\_$$

Advanced Mathematics (Module 1)

6. Evaluate the limit:

$$\lim_{x\to\infty} \sqrt{x}\left(\sqrt{x+5}-\sqrt{x+1}\right) = \_\_\_\_\_$$

7. The derivative of the function

$$f(x) = x^x \quad \text{is} \quad \_\_\_\_\_.$$

8. Tangent line of the circle $x^2 + y^2 = 25$ at the point $(4, 3)$ is expressed by an equation _____.

9. Evaluate the limit:

$$\lim_{x\to\frac{\pi}{3}} \frac{\sin x - \frac{\sqrt{3}}{2}}{x - \frac{\pi}{3}} = \_\_\_\_\_$$

10. Evaluate the limit:

$$\lim_{x\to 1} \frac{\sqrt{5-x}-2}{x-1} = \_\_\_\_\_$$

11. Evaluate the limit:

$$\lim_{x\to 1} \frac{x^7-1}{x^5-1} = \_\_\_\_\_$$

12. Given a curve described by the parametric equation

$$\begin{cases} x = \sin t \\ y = \cos t \end{cases},$$

the slope of the tangent line at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is _____.

13. The n-th order derivative of the function

$$f(x) = \frac{1}{x^2 - 3x + 2}$$

is given by _____.

14. Let $f(x)$ and $g(x)$ be two functions. Supposing that $f'(0) = 1,\ g'(0) = 3$ and $f(0) = g(0) = 0$, evaluate the limit

$$\lim_{x \to 0} \frac{f(x) - g(x)}{x} = \_\_\_\_\_.$$

15. The n-th order derivative of $\cos x$ is _____.

16. The n-th order derivative of $x^2 \sin x$ is _____.

17. Given that

$$\lim_{x \to 2} \frac{x^2 - ax - b}{x^2 - x - 2} = 4,$$

find the values of a and b: $a =$ _____, $b =$ _____.

Section B: Extended Answer Questions

This section is worth a total of **15** marks, and the marks for each question are as shown.

18. (**5** marks) Prove that the equation $x^2 - \sin x - 2 = 0$ has at least one root in the interval $(0, 2)$.

19. (**10** marks) Determine the values of the constant λ such that

$$\lim_{x \to \infty} \frac{\lambda x^2}{1 - x^2} = 2.$$