



Beijing-Dublin International College



SEMESTER I FINAL EXAMINATION – 2020/2021

School of Mathematics and Statistics
BDIC1029J & BDIC1025J Maths 1 (Advanced Mathematics)

HEAD OF SCHOOL: Wenying WU
MODULE COORDINATOR: Yanru PING
OTHER EXAMINERS: Yuehong FENG, Rong YANG

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **5** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

Section A: Gap-filling Questions

This section is worth a total of **75** marks, with each question worth **5** marks.

1. Evaluate

$$\lim_{t \rightarrow 0} \frac{\ln(1+t) - t}{t^2} = \underline{\hspace{2cm}}.$$

2. Find the limit

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = \underline{\hspace{2cm}}.$$

3. Find the limit

$$\lim_{x \rightarrow 0} \frac{\sqrt[5]{x \arctan x + 1} - 1}{x^2} = \underline{\hspace{2cm}}.$$

4. Let $f(x)$ be the function

$$f(x) = \frac{1}{(x+1)(x+2)}.$$

Find the higher order derivative $f^{(2020)}(x) = \underline{\hspace{2cm}}.$

5. Let $f(x)$ be the function

$$f(x) = \ln \sin x.$$

Find the differential $df(x) = \underline{\hspace{2cm}}.$

Advanced Mathematics (Module 1)

6. Find the limit

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} = \underline{\hspace{2cm}}.$$

7. Given

$$\lim_{x \rightarrow 0} \frac{\ln [1 + h(x)]}{\tan x} = 5,$$

find the limit

$$\lim_{x \rightarrow 0} \frac{h(x)}{x} = \underline{\hspace{2cm}}.$$

8. Given

$$\lim_{x \rightarrow \infty} \left(\frac{x^2}{1+x} + ax + b \right) = 0,$$

evaluate $a = \underline{\hspace{2cm}}$, and $b = \underline{\hspace{2cm}}$.

9. Given

$$\lim_{x \rightarrow 0} \frac{\tan x}{\sin ax} = 2,$$

evaluate $a = \underline{\hspace{2cm}}$.

10. Given

$$y = x \sin 2x,$$

find the higher order derivative $y^{(2020)} = \underline{\hspace{2cm}}$.

11. If the following function is continuous at $x = 0$,

$$f(x) = \begin{cases} (1 + \sin x)^{\frac{2}{x}}, & \text{if } x \neq 0; \\ A, & \text{if } x = 0, \end{cases}$$

try to determine the value of $A = \underline{\hspace{2cm}}$.

Advanced Mathematics (Module 1)

12. Given

$$\begin{cases} x = te^t, \\ e^t + e^y = 2, \end{cases} \quad t \text{ being a parameter, } t \in \mathbb{R},$$

evaluate

$$\frac{dy}{dx} = \underline{\hspace{2cm}}.$$

13. For

$$y = \left(\frac{x}{x+1} \right)^x,$$

try to find $\frac{dy}{dx} = \underline{\hspace{2cm}}.$

14. Evaluate

$$\lim_{x \rightarrow 1} \left[\frac{1}{x-1} - \frac{3}{(x^3-1)} \right] = \underline{\hspace{2cm}}.$$

15. Given $f(x) = x + \sin x$, $x \in [0, 2\pi]$, the interval(s) of concave-up for $f(x)$ is/are _____,
and the point(s) of inflection is/are _____.

Advanced Mathematics (Module 1)

Section B: Extended Answer Questions

This section is worth a total of **25** marks. The marks of every question is as shown.

- 16.** (*8 marks*) Suppose $f(x)$ is a continuous function over the interval $[0, 2]$. $f(x)$ is derivable on the point $(0, 2)$, with evaluation $f(2) = 0$.

Try to prove that there exists at least one number $\xi \in (0, 2)$, such that

$$f'(\xi) = -\frac{3f(\xi)}{\xi}.$$

- 17.** (*10 marks*) Given that a function

$$y = f(x) = ax^3 + bx^2 + cx + d$$

has an extreme value $y = 0$ occurring at $x = 0$, and $(1, 1)$ is an inflection point on the curve $y = f(x)$.

Try to determine the values of a , b , c and d .

- 18.** (*7 marks*) Prove

$$xe^x + ye^y \geq (x + y)e^{\frac{x+y}{2}}, \quad \forall x \geq y > -2.$$

Advanced Mathematics (Module 1)

USEFUL FORMULAE

$$\frac{d}{dx}C = 0$$

$$\frac{d}{dx}x^\mu = \mu x^{\mu-1}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}a^x = a^x \ln a$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\log_a x = \frac{1}{x \ln a}$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\operatorname{arccot} x = -\frac{1}{1+x^2}$$

Glossary

Concave-up/down	凹/凸
Curve	曲线
Decreasing	递减
Derivable	可导的
Derivative	导数
Differentiable	可微分的
Differential	微分
Discontinuity	不连续
Higher order derivative	高阶导数
Horizontal asymptote	水平渐近线
Infinitesimal	无穷小量
Interval	区间
Local maximum	极大值
Logarithmic differentiation	对数求导法
Minimum	最小值
Monotonic increasing	单调递增
Point of inflection	拐点
Tangent line	切线