Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION – 2021/2022

School of Mathematics and Statistics

BDIC1029J & BDIC1040J Maths 1 (Advanced Mathematics)

HEAD OF SCHOOL: Wenying WU
MODULE COORDINATORS: Bin ZHENG, Yanru PING
OTHER EXAMINERS: Rong YANG, Nicholas A. HOUSTON

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID: _____**

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ (Signature)

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **4** pages, with a full score of 100 marks.

You are required to use only the provided **Examination Book** for answers.

SECTION A — Brief Answer Questions

This section is worth a total of **80** marks, with each question worth **5** marks.

1. Evaluate the limit

$$\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 2x - 3} = \_\_\_\_\_.$$

2. Evaluate the limit

$$\lim_{x \to +\infty} (\sqrt{x^2 + 2x} - x) = \_\_\_\_\_.$$

3. Evaluate the limit

$$\lim_{x \to 0} \frac{\sqrt{\cos x} - 1}{x} = \_\_\_\_\_.$$

4. Let $f(x) = xe^x$, compute the 20-th derivative of f.

$$f^{(20)}(x) = \_\_\_\_\_.$$

5. Find the differential for the function $f(x) = 3\cos^2 x$, $df =$ _____. Use the differential to approximate the change in $f(x)$ when we have a small change in x, $dx = 0.01$, at $x = \pi/4$, and estimate $f(\pi/4 + 0.01) \approx$ _____.

6. Evaluate the limit

$$\lim_{x\to+\infty} x^{\frac{1}{1+\ln x}} = \text{\_\_\_\_\_}.$$

7. If $f(9) = 1, f'(9) = 3, g(1) = 4, g'(1) = -2$, and $h(x) = x^{3/2}g(f(x))$, then

$$h'(9) = \text{\_\_\_\_\_}.$$

8. Given $f(x) = \frac{2x^2}{x-3x^2}$, then the equation of the horizontal asymptote of the curve $y = f(x)$ is $y =$ _____, whereas the equation of the vertical asymptote is $x =$ _____.

9. Evaluate the limit

$$\lim_{x\to 0} \frac{\tan^2(x + \frac{\pi}{4}) - \tan^2(\frac{\pi}{4})}{x} = \text{\_\_\_\_\_}.$$

10. Given $y = e^{\frac{-x^2}{2}}$, compute the first, second, and third derivatives of y.

$$y' = \text{\_\_\_\_\_}, \quad y'' = \text{\_\_\_\_\_}, \quad y''' = \text{\_\_\_\_\_}.$$

11. Evaluate the limit

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{e^t-1}\right)=\_\_\_\_\_.$$

12. Given

$$\begin{cases} x &= 3t^2+1, \\ y &= t^3-2t^2, \end{cases} \quad \text{where } t\in\mathbb{R} \text{ is a parameter.}$$

Compute $\frac{dy}{dx}=$ _____, $\frac{d^2y}{dx^2}=$ _____.

13. Given $y=(\cos x)^{\sin x}$, then $\frac{dy}{dx}=$ _____.

14. Given $y-e^{xy}=0$, then $y'=$ _____.

15. Find the values of the constants a and b, such that the function

$$f(x)=\begin{cases} ax+b & x<0 \\ x^3+2x+1 & x\geq 0, \end{cases}$$

is differentiable at $x=0$. (Hint: Find $a=$ _____, $b=$ _____.)

16. Given $f(x)=2x^3-3x^2-12x+1$, defined on a closed interval $x\in[-2,0]$. Its absolute maximum value is _____, absolute minimum value is _____.

SECTION B — Extended Answer Questions

Write your answers on the **Examination Book** provided.

This section is worth a total of **20** marks, with each question worth 5 marks.

17. Suppose $f(x)$ is continuous and differentiable for all real numbers. If $-1 \leq f'(x) \leq 3$ and $f(5) = 6$, what is the largest $f(x)$ can be at $x = 1$?

18. Prove the identity: $\arcsin x + \arccos x = \frac{\pi}{2}$, where $x \in [-1, 1]$.

19. Evaluate the limit

$$\lim_{x \to 0} \frac{x - \sin x}{x^3}$$

by the L'Hospital's Rule.

20. Prove the inequality: $1 + 2\ln x \leq x^2$, for $x > 0$.

Glossary

Absolute maximum	最大值
Absolute minimum	最小值
Approximate	近似
Continuous	连续
Curve	曲线
Derivative	导数
Differentiable	可微分的
Differential	微分
Estimate	估计
Horizontal asymptote	水平渐近线
Identity	恒等式
Inequality	不等式
Interval	区间
Limit	极限
Parameter	参数
Vertical asymptote	铅垂渐近线