Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION – 2017/2018

School of Mathematics and Statistics

BDIC1029J & BDIC1025J Maths 1 (Advanced Mathematics)

HEAD OF SCHOOL: Wenying WU
MODULE COORDINATOR: Yanru PING
OTHER EXAMINER: Yuehong FENG

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **4** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

Section A: Fill-in-the-blank Questions

This section is worth a total of **80** marks, with each question worth **5** marks.

1. Given

$$\lim_{x\to 0}\frac{\sin x}{\sin ax}=5,$$

evaluate $a=$ _____.

2. Given

$$\lim_{x\to 0}\frac{\ln\left[1+f(x)\right]}{\sin x}=3,$$

find the limit

$$\lim_{x\to 0}\frac{f\left(x\right)}{x}=\text{\_\_\_\_\_}.$$

3. Find the limit

$$\lim_{x\to 2}\frac{\sqrt{x^2-4x+5}-1}{(x-2)^2}=\text{\_\_\_\_\_}.$$

4. Find the limit

$$\lim_{x\to 0}\frac{e^{\cos x-1}-1}{x^2}=\text{\_\_\_\_\_}.$$

5. Find the limit

$$\lim_{x\to\infty}\left(\frac{x^2+1}{x^2+2}\right)^{x^2}=\text{\_\_\_\_\_}.$$

6. Find the limit

$$\lim_{n\to\infty} \sqrt[n]{3^n + 5^n} = \_\_\_\_\_.$$

7. Find the limit

$$\lim_{x\to 0} \frac{x - \pi}{\sin x} = \_\_\_\_\_$$

8. Given

$$\lim_{x\to\infty} \frac{x^5 + 4x^3 + 2x - 7}{1 - x^5} = b,$$

evaluate $b =$ _____. Then find the equation of the horizontal asymptote of the curve

$$y = \frac{x^5 + 4x^3 + 2x - 7}{1 - x^5}.$$

9. Evaluate

$$\lim_{x\to 0} \frac{e^{3x} - e^{5x}}{x} = \_\_\_\_\_$$

10. Given

$$y = x \cdot \cos 3x,$$

find the higher order derivative $y^{(8)} =$ _____.

11. Let $f(x)$ be the function

$$f(x) = \frac{x}{x + 1}.$$

Find the higher order derivative $f^{(2018)}(x) =$ _____.

12. Given

$$\begin{cases} x = 2t + 1, \\ y = \arctan t, \end{cases} \qquad t \text{ being a parameter, } \ t \in \mathbb{R},$$

evaluate

$$\frac{dy}{dx} = \_\_\_\_\_, \qquad \frac{d^2y}{dx^2} = \_\_\_\_\_.$$

13. Use logarithmic differentiation to find $\dfrac{dy}{dx}$, where

$$y = \frac{x^2\sqrt{1-x^3}}{(x^2+1)^3}.$$

14. Given

$$y = \frac{x-5}{\sqrt[3]{x^2+2}},$$

find

$$\frac{dy}{dx} = \_\_\_\_\_.$$

15. Evaluate

$$\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right),$$

16. Let $f(x)$ be the function

$$f(x) = \sin\ln x.$$

Find the differential $df(x) = \_\_\_\_\_$.

Advanced Mathematics (Module 1)

Section B: Extended Answer Questions

This section is worth a total of **20** marks, with each question worth **5** marks.

17. Let $y(x)$ be the function

$$y = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} + \sin(x-2)\sin\frac{1}{x-2}.$$

Find all the discontinuous point(s) of $y(x)$, and determine the type of discontinuity for each point.

18. Prove that the equation $\sin x = x$ has only one root.

19. Given $f(x) = 2x^3 - 9x^2 + 12x + 1$, determine its monotonic increasing interval and monotonic decreasing interval, and find its local maximum value and local minimum value.

20. Suppose $f(x)$ is a continuous function over the interval $[0,1]$. $f(x)$ is derivable on the point $(0,1)$, with evaluation $f(1) = 0$.

Try to prove that there exists at least one number $\xi \in (0,1)$, such that

$$f'(\xi) = -\frac{3f(\xi)}{\xi}.$$

Glossary

Auxiliary function	辅助函数
Curve	曲线
Decreasing	递减
Derivative	导数
Differentiable	可微分的
Differential	微分
Discontinuity	不连续
Higher order derivative	高阶导数
Horizontal asymptote	水平渐近线
Infinitesimal	无穷小量
Local maximum	局部最大值
Logarithmic differentiation	对数求导法
Minimum	最小值
Monotonic increasing	单调递增
Tangent line	切线