Beijing-Dublin International College

SEMESTER 2 RESIT EXAMINATION – 2020/2021

School of Mathematics and Statistics
BDIC1029J & BDIC1025J Maths 1 (Advanced Mathematics)

HEAD OF SCHOOL: Wenying WU
MODULE COORDINATOR: Yanru PING
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Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **4** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

Section A: Fill-in-the-blank Questions

This section is worth a total of **75** marks, with each question worth **5** marks.

1. Evaluate

$$\lim_{t\to 0}\frac{e^t-t-1}{t^2}=\_\_\_\_\_$$

2. Find the limit

$$\lim_{x\to 0} x\cdot\sin\frac{1}{x}=\_\_\_\_\_$$

3. Find the limit

$$\lim_{x\to 0}\frac{\sqrt{2x^2+1}^5-1}{x^2}=\_\_\_\_\_.$$

4. Let $f(x)$ be the function

$$f(x)=\ln(x+2).$$

Find the higher order derivative $f^{(2021)}(x)=\_\_\_\_\_$.

5. Given $f'(1)=2$,then

$$\lim_{h\to 0}\frac{f(1-2h)-f(1)}{h}=\_\_\_\_\_$$

6. Find the limit

$$\lim_{n\to\infty}\sqrt[n]{2^n+1}=\_\_\_\_\_.$$

7. Given

$$\lim_{x\to 0}\frac{\ln\left[1+h(x)\right]}{\tan 2x}=5,$$

find the limit

$$\lim_{x\to 0}\frac{h\left(x\right)}{x}=\_\_\_\_\_.$$

8. Given

$$\lim_{x\to -1}\frac{x^2-3x-a}{1+x}=b,$$

evaluate $a=\_\_\_\_\_, b=\_\_\_\_\_$.

9. The horizontal asymptote of the curve $y=\dfrac{1-3x-x^2-x^3}{x^3-6}$ can be expressed by an equation _____.

10. Given

$$y=x\cdot\cos 2x,$$

find the higher order derivative $y^{(2021)}=\_\_\_\_\_$.

11. The function

$$f(x)=\begin{cases}(1+\arctan x)^{\frac{1}{x}} & x\neq 0\\ A & x=0\end{cases}$$

is continuous everywhere, then $A=\_\_\_\_\_$.

12. Given a curve with parametric equations

$$\begin{cases}x=\cos^2 t,\\ y=\sin^2 t\end{cases}\qquad t \text{ being a parameter, } t\in\mathbb{R},$$

, and a point P at the curve corresponding $t=\frac{\pi}{6}$, then the equation of the tangent line at the point P can be expressed as _____.

13. Find $\frac{dy}{dx}$, where

$$y = x^{x+1}.$$

14. Evaluate

$$\lim_{x \to 1} \left(\frac{1}{x-1} - \frac{2}{(x^2-1)} \right),$$

15. $y = f(x)$ is determined by the equation $x^3 + y^3 - \tan x + 6y = 0$, the $\frac{dy}{dx}|_{x=0} =$ _____

Advanced Mathematics (Module 1)

Section B: Extended Answer Questions

This section is worth a total of **25** marks.

16. (10 marks) Suppose $f(x)$ is a continuous function over the interval $[0,1]$. $f(x)$ is derivable on the point $(0,1)$, with evaluation $f(0) = f(1) = 1$.

Try to prove that there exists at least one number $\xi \in (0,1)$, such that

$$f'(\xi) = -\frac{2f(\xi)}{\xi}.$$

17. (8 marks) Given that function $y = x^3 - 3x^2 - 24x + 7$, find its extreme values.

18. (7 marks)Prove that $e^x + e^y \geq 2\, e^{\frac{(x+y)}{2}}$ for any real numbers x, y

Glossary

derivable	可导的
Curve	曲线
Decreasing	递减
Derivative	导数
Differentiable	可微分的
Differential	微分
Discontinuity	不连续
Higher order derivative	高阶导数
Horizontal asymptote	水平渐近线
Infinitesimal	无穷小量
Local maximum	极大值
Logarithmic differentiation	对数求导法
Minimum	最小值
Monotonic increasing	单调递增
Tangent line	切线
Point of inflection	拐点
Concave up /down	凹/凸
Interval	区间

导数公式：

$(C)' = 0$

$(x^{\mu})' = \mu x^{\mu-1}$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x$

$(\cot x)' = -\csc^2 x$

$(\sec x)' = \sec x \tan x$

$(\csc x)' = -\csc x ctgx$

$(a^x)' = a^x \ln a$

$(e^x)' = e^x$

$(\log_a x)' = \dfrac{1}{x \ln a}$

$(\ln x)' = \dfrac{1}{x}$

$(\arcsin x)' = \dfrac{1}{\sqrt{1 - x^2}}$

$(\arccos x)' = -\dfrac{1}{\sqrt{1 - x^2}}$

$(\arctan x)' = \dfrac{1}{1 + x^2}$

$(arc \cot x)' = -\dfrac{1}{1 + x^2}$