



Beijing-Dublin International College



SEMESTER I RESIT EXAMINATION – 2017/2018

School of Mathematics and Statistics

BDIC1014J & BDIC1044J Linear Algebra

PRINCIPAL OF COLLEGE: Wenying WU

MODULE COORDINATOR: Xin LIU

OTHER EXAMINER: Jinru WANG

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____

UCD Student ID: _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.

No rough-work paper is to be provided for candidates.

1. (10 marks) Find an appropriate elementary matrix (适当的初等矩阵) for each of the following transformations (描述下列变换):

$$(a) \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 3 & 4 \\ 1 & 2 \\ 5 & 6 \end{pmatrix}; \quad (b) \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 4 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}; \quad (c) \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 5 & 6 \end{pmatrix}.$$

2. (10 marks) Evaluate the following determinant (计算行列式):

$$\det \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & 7 \\ 0 & 2 & 1 \end{pmatrix}.$$

3. (10 marks) Use the Cramer's rule (克莱默法则) to solve the following linear system (解方程组):

$$\begin{cases} 3x + y = 2, \\ 2x - y = 3. \end{cases}$$

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4. (10 marks) Find the inverse (矩阵的逆), A^{-1} , of the following matrix, in terms of the method of row reductions (行变换):

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix}.$$

5. (10 marks) Find the inverse of the following matrix, A^{-1} , in terms of the method of adjoint matrix (伴随矩阵), $\text{adj } A$:

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix}.$$

6. (10 marks) Find the rank (矩阵的秩) of the following matrix:

$$\text{rank} \begin{pmatrix} 2 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 1 & 1 & 4 \\ 0 & 4 & 2 & 4 \end{pmatrix}.$$

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In Questions 7—9 below, consider the following matrix:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

7. (20 marks) Find the eigenvalues (本征值) of A , as well as the corresponding eigenvector (本征向量) for each eigenvalue.

8. (10 marks) Diagonalize the matrix A (对角化矩阵).

9. (10 marks) Prove (证明) the following fact:

$$A^n = 3^{n-1}A.$$