Beijing-Dublin International College

SEMESTER I RESIT EXAMINATION – 2016/2017

School of Mathematics and Statistics

BDIC1014J & BDIC1022J Linear Algebra

HEAD OF SCHOOL: Gary McGuire
MODULE COORDINATOR: Xin LIU

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

1. (30 marks) Consider the following system of linear equations:

$$\begin{cases} x & & -2z & = & 1, \\ 2x & +y & -z & = & 2, \\ -x & & +z & = & -1. \end{cases}$$

(a) Use the method of row operations（初等行变换）to solve this linear system.

(b) Use the method of $\mathbf{x} = A^{-1}\mathbf{b}$ （逆矩阵方法）to solve this system.

(c) Use the method of Cramer's rule（克莱姆法则）to solve this system.

2. **(15 marks)** True & False（判断对错，注明理由）:

Judge the correctness of the following statements (by circling the given words *TRUE* or *FALSE*), and justify your choice (i.e., explain your reason to do the choice).

(a) Let A be a 3×3 matrix. Then **TRUE** **FALSE**

$$\det(-A) = \det A.$$

Reason:

(b) Let A and B be two $n \times n$ matrices. **TRUE** **FALSE**

$$(A+B)(A-B) = A^2 - B^2.$$

Reason:

(c) Let A be an $n \times n$ invertible matrix（可逆矩阵）, and I the $n \times n$ identity（恒等矩阵）. **TRUE** **FALSE**

$$(A-I)\left(A^{-1}+I\right) = A - A^{-1}.$$

Reason:

(d) Let A be a matrix and λ its eigenvalue（本征值）, with $\mathbf{v}$ as the corresponding eigenvector（本征矢量）, i.e.,

$$A\mathbf{v} = \lambda\mathbf{v}.$$

Then λ^m is an eigenvalue of the matrix A^m, with $m = 1, 2, 3, \cdots$. **TRUE** **FALSE**

Reason:

3. (35 marks) Diagonalize the matrix（对角化矩阵）

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

and compute A^{2017}.

4. (20 marks) Fill in the blanks.

(a) Find the appropriate elementary matrix（初等矩阵）to realize the type-I row operation（第一类行变换）

$$\begin{pmatrix} \quad & \quad & \quad & \quad \\ \\ \\ \\ \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 4 \\ 3 \\ 2 \end{pmatrix}.$$

(b) Find the appropriate elementary matrix to realize the type-II row operation（第二类行变换）

$$\begin{pmatrix} \quad & \quad & \quad & \quad \\ \\ \\ \\ \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 2 \\ 9 \\ 4 \end{pmatrix}.$$

(c) Find the appropriate elementary matrix to realize the type-III row operation（第三类行变换）

$$\begin{pmatrix} \quad & \quad & \quad & \quad \\ \\ \\ \\ \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 5 \\ 3 \\ 4 \end{pmatrix}.$$