LINEAR ALGEBRA

(BDIC1014J / BDIC1044J)

Lecturers: *Xin LIU, Nicholas A. HOUSTON, Changjing ZHUGE*

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Beijing-Dublin International College (BDIC), Beijing University of Technology (BJUT)

Tutorial 3

VECTOR PART — Chapter 3

1. Find parametric vector, parametric scalar and Cartesian equations of the line passing through P in the direction of $\boldsymbol{v}$ in each of the following cases:

   (i) $P = (1, 0, -1),\ \boldsymbol{v} = 2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$; (ii) $P = (-6, 5, 2),\ \boldsymbol{v} = 2\boldsymbol{i} - 5\boldsymbol{k}$;

   (iii) $P = (0, 1, -1),\ \boldsymbol{v} = \boldsymbol{i} + 2\boldsymbol{j}$; (iv) $P = (2, 3, -3),\ \boldsymbol{v} = -\boldsymbol{i}$.

2. Find parametric vector, parametric scalar and Cartesian equations of the line passing through P and Q in each of the following cases:

   (i) $P = (-4, 3, 5), Q = (-2, 4, -1)$; (ii) $P = (0, 5, 0), Q = (5, 0, -5)$.

3. Find vector and Cartesian equations of the plane containing P having normal vector n in each of the following cases:

   (i) $P = (4, -1, 0),\ \boldsymbol{n} = 3\boldsymbol{i} + \boldsymbol{j} - 4\boldsymbol{k}$; (ii) $P = (7, 5, -3),\ \boldsymbol{n} = 2\boldsymbol{i} + \boldsymbol{k}$;

   (iii) $P = (0, 0, -9),\ \boldsymbol{n} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$; (iv) $P = (-6, 5, 6),\ \boldsymbol{n} = \boldsymbol{j}$.

4. Find a Cartesian equation for the plane containing $P = (6, 7, -2),\ Q = (0, -8, 11),\ R = (14, -3, 9)$.

5. The planes $x + y + z = 2$ and $x - y + 3z = 0$ intersect in a line $\mathcal{L}$.

   (i) Find a point on $\mathcal{L}$.

   (ii) Use cross products to find a vector pointing in the direction of $\mathcal{L}$.

   (iii) Write down parametric scalar and Cartesian equations for $\mathcal{L}$.

6. Two lines in space are skew if they are not parallel and do not intersect. The following lines are not parallel. Show that they are not skew, by finding their point of intersection:

$$\mathcal{L}_1 : \boldsymbol{r} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k} + t\,(3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k})\,; \qquad \mathcal{L}_2 : \boldsymbol{r} = 6\boldsymbol{i} - 6\boldsymbol{j} + \boldsymbol{k} + s\,(-7\boldsymbol{i} + 5\boldsymbol{j} - 6\boldsymbol{k})\,.$$

7. Find the distance from $P(3, 0, -1)$ to the plane $\mathcal{P}$ described by the equation

$$4x + 2y - z = 6.$$

Find the closest point to P which lies on $\mathcal{P}$.

8. For each of (i)–(vii), find two matching descriptions from (a)–(n).

(i) line containing $(0, 0, 0)$ in the direction of $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$

(ii) line containing $(-1, 2, -1)$ in the direction of $-\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$

(iii) line containing $(-1, 2, -1)$ and $(0, 0, -2)$

(iv) plane containing $(0, 0, 0)$ with normal vector $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$

(v) plane containing $(-1, 2, -1)$ with normal vector $-\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$

(vi) plane containing $(-1, 2, -1)$, $(0, 0, -2)$ and $(1, 3, 3)$

(vii) plane containing $(-1, 2, -1)$, $(0, 0, -2)$ and $(1, 3, 2)$

(a) $x + y + z = 0$

(b) $x = y = z$

(c) $x + y - z = 2$

(d) $x + 1 = \frac{y-2}{-2} = \frac{z+1}{-2}$

(e) $7x + 6y - 5z = 10$

(f) $x + 1 = \frac{y-2}{-2} = \frac{z+1}{-1}$

(g) $x - 2y - 2z = -3$

(h) $(\boldsymbol{r} + 2\boldsymbol{k}) \cdot (7\boldsymbol{i} + 6\boldsymbol{j} - 5\boldsymbol{k}) = 0$

(i) $\boldsymbol{r} = \boldsymbol{i} - 2\boldsymbol{j} - 5\boldsymbol{k} + t\,(\boldsymbol{i} - 2\boldsymbol{j} - 2\boldsymbol{k})$

(j) $(\boldsymbol{r} + 2\boldsymbol{k}) \cdot (\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}) = 0$

(k) $\boldsymbol{r} \cdot (\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) = 0$

(l) $(\boldsymbol{r} + 3\boldsymbol{i}) \cdot (\boldsymbol{i} - 2\boldsymbol{j} - 2\boldsymbol{k}) = 0$

(m) $\boldsymbol{r} = \boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k} + t\,(\boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k})$

(n) $\boldsymbol{r} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k} - t\,(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$.

9. Find Cartesian equations of the line passing through $(1, 0, -2)$ and perpendicular to the plane $3x - 4y + z = 6$.

10. Verify that the line

$$\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$$

is parallel to the plane $4x + 4y - 5z = 14$.

11. Find the Cartesian equation of the plane containing $(1, 1, 1)$ and the line

$$\frac{x-4}{-2} = y + 3 = \frac{z-1}{3}.$$

12. Find the angle θ between two lines $l_1 : x - 1 = \frac{y}{-4} = z + 3$ and $l_2 : \frac{x}{2} = \frac{y+2}{-2} = \frac{z}{-1}$.

13. Find the equation of a line l that passes through a fixed point $P(4, -2, 8)$ and is perpendicular to the plane $2x - 3y + z - 4 = 0$.

14. Determine the equation of a plane that passes through three points $P_1(1, 3, 5)$, $P_2(2, -1, 3)$ and $P_3(-3, 2, -6)$.

15. Find the distance from $P(2, 1, 1)$ to the line $\mathcal{L}$ given by the equations

$$x - 1 = \frac{y-1}{3} = \frac{z+4}{-1}.$$

Find the closest point to P lying on $\mathcal{L}$.

LINEAR ALGEBRA PART — Chapter 4: Matrices and systems of equations

16. Solve the following systems of equations by assigning $z = t$ and expressing x and y in terms of the parameter t:

$$\text{(i)} \begin{cases} x - 2z = 4 \\ y + z = 2 \end{cases} \qquad \text{(ii)} \begin{cases} x + 2y + 3z = 0 \\ y - 2z = -1 \end{cases}$$

17. Find parametric scalar equations for the line of intersection of the two planes in each of the following cases:

$$\text{(i)} \begin{cases} x + y + z = 2 \\ x - y + 3z = 0 \end{cases} \qquad \text{(ii)} \begin{cases} -3x + 2y + 7z = 1 \\ 5x - 3y - 2z = -2 \end{cases}$$

18. Solve the following homogeneous systems of equations:

$$\text{(i)} \begin{cases} x + 2y + 3z = 0 \\ 3x + 2y + z = 0 \end{cases} \qquad \text{(ii)} \begin{cases} -x + y + z - w = 0 \\ 2x + z + w = 0 \\ x - 2y + z + 3w = 0 \end{cases}$$

19. Give a very brief reason why a homogeneous system can never be inconsistent.

20. Solve the following systems of equations:

(i) $\begin{cases} x + 2y + 7z = 5 \\ x + y + 4z = 3 \\ 2x + 3y + 11z = 7 \end{cases}$ (ii) $\begin{cases} x + 2y + z - w = 4 \\ 2x + 4y - z + 4w = -1 \\ -x - 2y + 2z - 5w = 5 \end{cases}$

21. A cubic polynomial in x takes the value -2 when $x = 1$ and the value -10 when $x = -1$. Its derivative takes the value 0 when $x = 1$ and the value 12 when $x = -1$. Find the polynomial.

22. Find the values of λ such that the following system (i) is inconsistent; (ii) has in finitely many solutions; (iii) has a unique solution:

$$\begin{cases} x - 3z = -3, \\ -2x - \lambda y + z = 2, \\ x + 2y + \lambda z = 1. \end{cases}$$